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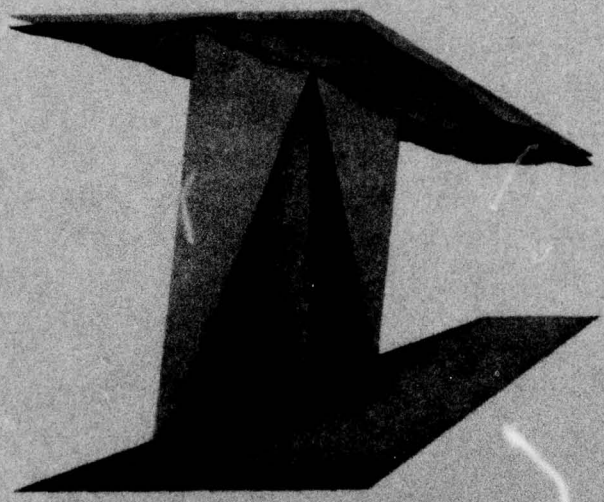
"INVERT", A COMPUTER PROGRAM FOR OBTAINING D-REGION ELECTRON DENSITY PROFILES FROM VLF REFLECTION COEFFICIENTS.

Interim Report No. 782

D.G. Morfitt and C.H. Shellman



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INVERT is a FORTRAN computer program designed to study the feasibility of obtaining electron density distributions of the D-region of the ionosphere. These electron density profiles would be derived from measurements of ionospheric reflection coefficients. The radio propagation frequencies would be limited to the VLF band, particularly 3-20 kHz. This report contains a discussion of the analytical approach taken in INVERT, a FORTRAN listing of the program, instructions for using the program and some sample calculations using simulated data. The program has not been used with real data.

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TABLE OF CONTENTS

TABLE OF CONTENTS Continued

ABSTRACT

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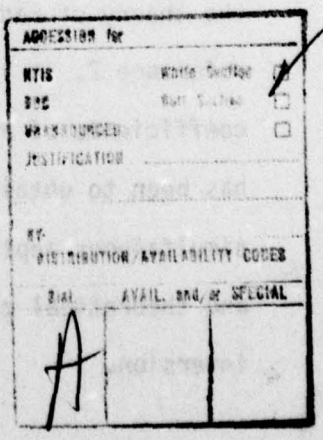


I. INTRODUCTION

The nature of the lowest ionosphere, the D-region, has been the objective of Government sponsored research for more than a decade. The principal interest stems from the global requirement for reliable strategic communications at very low radio frequencies (VLF 3-30 kHz).

Numerical methods are presently available for calculating VLF systems performance capabilities if the height distributions of the electron density and the collision frequency of the lower ionosphere are known. The problem is, however, to develop a procedure that will result in a reasonable representation of these required profiles.

Radio sounding at VLF is an often-used method for exploring the D-region. This region is considered to range from 50 km to about 100 km above the earth's surface. Because of the low-electron densities characteristic of these ionospheric heights, usual methods, such as high frequency (hf) ionosounds provide no useful information. For this same reason, data from rocket soundings also provide little detail at these heights.

The measured data parameters, acquired from VLF ionospheric sounders, are the ionospheric reflection coefficients. With special sounding techniques, such as described in reference 1, it is possible to obtain reflection coefficient data simultaneously at many frequencies over the VLF band. Also, since the theory of radio wave reflection from the ionosphere is well developed, reference 2, it is possible to calculate the characteristic reflection coefficients of radio waves propagated in models of the ionosphere. The goal has been to obtain the height distributions of the ionospheric factors by the simultaneous application of these two procedures, (i.e. sounder measurements and theoretical calculations). Such an approach would be known as profile inversion.

Early work in ionospheric inversion used trial-and-error techniques to deduce profiles. That is, values for the ionospheric properties were assumed and the ionospheric reflection coefficients calculated; these results were then compared with experimental values, and if the agreement was not satisfactory, the calculations were performed again with different values used for the ionospheric properties. The goodness of fit between calculations and experimental data were determined by a subjective estimate of the investigator.

The approach taken in the INVERT computer program is to attempt an analytical determination of the ionospheric profiles without the use of trial-and-error. The procedure begins with ionospheric reflection coefficient data and seeks to deduce the required electron density profile by finding the rate of change of the reflection coefficients with respect to changes at each point in the profile. Ideally the end result of the INVERT inversion procedure will be a profile which contains all the detail justified by the data, but which does not contain spurious data which would represent fitting to error in the data.

The fundamental ideas of the INVERT inversion technique have been previously described in reference 3. For this reason, several references will be made to that report rather than to reproduce the material here.

This report is divided into six major sections other than the introduction. Section II presents a discussion of the theory used in the inversion process. Section III traces the execution of the INVERT procedure. Section IV gives a description of the INVERT computer program. Section V discusses execution of INVERT for simulated data while Section VI is for measured data. The Appendices A, B, and C give FORTRAN listings of the computer programs.

The notation used throughout this report is as described below, where in general the subscript convention used is that j and j' refer to the "n" layers of the electron density profile while i and i' refer to the "m" data parameters. Note that most equations are written so that subscripted variables are thought of as matrices or vectors.

The notation is:

Square matrix:

$$\underset{\sim}{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} = \begin{matrix} & & j' \rightarrow \\ j & \downarrow & \end{matrix} \begin{pmatrix} & & \\ & a_{jj'} & \\ & & \end{pmatrix} \qquad (A)$$

Rectangular matrix:

$$\underset{\sim}{B} = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nm} \end{bmatrix} = \begin{matrix} & & i \rightarrow \\ j & \downarrow & \end{matrix} \begin{bmatrix} & & \\ & b_{ji} & \\ & & \end{bmatrix} \qquad (B)$$

Column vector:

$$\vec{C} = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = \begin{matrix} j \\ \downarrow \end{matrix} \begin{pmatrix} \\ c_j \\ \end{pmatrix} \qquad (C)$$

Row vector:

$$\vec{D} = \overbrace{d_1 \cdots d_n}^{} = \overbrace{d_{j'}}^{j' \rightarrow} \qquad (D)$$

II. INVERSION THEORY

The inversion problem is to begin with reflection coefficient data and deduce the responsible electron density profile.

A. Reflection Coefficient Data

Very low frequency (VLF) reflection coefficient data may be obtained from steep incidence sounders. Reflection of a VLF wave at near-vertical incidence is described by the reflection matrix:

$$\underset{\sim}{R} = \begin{pmatrix} {}_{\parallel}R_{\parallel} & {}_{\perp}R_{\parallel} \\ {}_{\parallel}R_{\perp} & {}_{\perp}R_{\perp} \end{pmatrix} \tag{1}$$

where the first subscript refers to the polarization of the up going wave with respect to the plane of incidence and the second subscript refers to the polarization of the down coming wave. Note that the matrix elements are complex numbers.

The reflection coefficient ratio used in the inversion procedure is given in equation (66) of reference 3 as:

$$R(\omega_i) = \left\{ \frac{- {}_{\perp}R_{\perp}(\theta) + j\ {}_{\perp}R_{\parallel}(\theta)}{- {}_{\perp}R_{\perp}(\theta) - j\ {}_{\perp}R_{\parallel}(\theta)} \right\}_i \tag{2}$$

where $i = 1, \ldots m$

The terms of equation (2) are further identified as:

ω_i -- The transmitted frequency.

m -- The total number of transmitted frequencies.

θ -- The incident angle of the radio wave on the ionosphere.

j -- $\sqrt{-1}$

$R(\omega_i)$ -- The value of the reflection ratio at the i-th frequency as obtained from data.

The reflection coefficients of equations (1) and (2) are functions of the earth's magnetic field. As the sounder site approaches the geomagnetic

equator, the cross term (i.e. $_{\perp}R_{\parallel}$) tends to zero and thus the reflection ratio, $R(\omega_i)$, becomes equal to one. When this situation occurs, the inversion technique yields no information. The most useful data for determining profiles is obtained, therefore, at locations with large magnetic dip angles (i.e. the magnetic poles).

B. Parameterization of the Electron Density Profile

The procedure is to represent the electron density profile by a series of short exponential segments. The common logs of the electron densities at the heights, z_j, where the segments join, are taken to be the unknown parameters of the profile and are denoted by:

$$\alpha_j = \alpha(z_j) = \log_{10} N_e(z_j) \tag{3}$$

where j = 1, 2, ... n.

and where N_e is the electron density (electrons/cc) at height z (km).

C. Computed Reflection Coefficients

The ionospheric reflection coefficients given in equation (1) may be computed for any given electron density profile by the "full-wave" method given in reference 4. These reflection coefficients are the result of the integration:

$$\underset{\sim}{R} = \int_b^a \left(\frac{d\underset{\sim}{R}}{dz}\right) dz \tag{4}$$

As discussed previously for measured data, a useful representation of the computed reflection coefficient is:

$$r(\omega_i) = \left\{\frac{-_{\perp}R_{\perp}(\theta) + j\ _{\perp}R_{\parallel}(\theta)}{-_{\perp}R_{\perp}(\theta) - j\ _{\perp}R_{\parallel}(\theta)}\right\}_i \tag{5}$$

i = 1, ... m

In this instance the reflection coefficients, ${}_{\perp}R_{\perp}$ and ${}_{\perp}R_{\parallel}$ are computed from the integration of equation (4) and thus "$r(\omega_i)$" is the computed value of the reflection coefficient parameter at the i-th frequency.

D. Fundamentals of the Inversion Procedure

The approach taken in searching for a best-fit electron density profile to data was originally presented in reference 5. The basic requirement is that the profile is to be linear (on a log scale) unless the data gives information to justify a given degree of detail. To carry out this approach, two functions are defined. The first is a measure of deviation of computed reflection coefficients from measured data, defined as:

$$s = \sum_{i}^{m} \left\{ (r_i - R_i)/\sigma_i \right\}^2 \qquad (6)$$

$$i = 1, \ldots m$$

where

m -- is the total number of propagation frequencies.

r_i -- are the computed values of the reflection coefficient ratio from equation (5).

R_i -- are the "measured" reflection coefficients in the form of equation (2).

σ_i -- are the uncertainties in the "measured" values. These are also defined as the "expected" errors.

The second function is a measure of detail, or curvature, in the profile and is defined as:

$$c = \int_{b}^{a} \left\{ \frac{d^2 \alpha(z)}{dz^2} \right\} dz \qquad (7)$$

where z is the height variable, $\alpha(z)$ is the common log of the electron density profile at height, z, and the integral is taken over the range of the the profile.

Equation (7) defines the entire profile curve, $\alpha(z)$. In practice $\alpha(z)$ can only be determined at a finite number of points, $(\alpha(z_j) = \alpha_j)$, and therefore the form of the profile is given by equation (3). For simplicity it will be assumed that the α_j's are equally spaced at intervals, Δz. The second derivative at z_j is then approximated by:

$$\frac{d^2\alpha}{dz} = \left[\left(\frac{\alpha_{j-1} - \alpha_j}{\Delta z}\right) - \left(\frac{\alpha_j - \alpha_{j+1}}{\Delta z}\right)\right] \Delta z \qquad (8)$$

In application, the curvature function "c" of equation (7) is expressed in terms of summation over profile segments. The equation becomes:

$$c \approx \left\{\sum_{j=2}^{n-1} \left(\alpha_{j-1} - 2\alpha_j + \alpha_{j+1}\right)^2\right\} (\Delta z)^{-3} \qquad (9)$$

where n is the number of points α_j.
The expression in brackets represents the second derivative at height z_j. The slab summation form of "c" approaches the integral of equation (7) as the slabs are made sufficiently thin.

Equation (9) can be written in matrix notation as:

$$c \approx (\Delta z)^{-3} \cdot \overbrace{\alpha_{j'}}^{j' \rightarrow} \cdot \underset{\downarrow}{j'}\left(C_{j'j}\right) \cdot \overset{j \rightarrow}{\underset{\downarrow}{j}}\left(\alpha_j\right) \qquad (10)$$

where the $C_{j,j}$ are elements of the matrix

$$\underset{\sim}{C} = \begin{pmatrix} 1 & -2 & 1 & & & & & & \\ -2 & 5 & -4 & 1 & & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & & 1 & -4 & 6 & -4 & 1 \\ & & & & & 1 & -4 & 5 & -2 \\ & & & & & & 1 & -2 & 1 \end{pmatrix} \tag{11}$$

The required profile is sought by an iterative technique in which the total curvature in the profile is allowed to increase in steps to improve the fit to data. Each step requires a perturbation-type solution in which the full-wave solution of reference 4 is analytically differentiated with respect to perturbations in the profile. The end result of the inversion procedure is a profile which contains all the detail justified by the data, but which does not contain spurious detail which would represent fitting to error in the data. There should be an optimum trade-off between deviation from data, represented by the value of s, and curvature in the profile, represented by the value of c. The trade-off may be represented by the condition:

$$c + \lambda s \rightarrow \min \tag{12}$$

where the value of λ must be chosen for the optimum condition.

The solution to equation (12) is obtained by differentiating with respect to α_j. This gives:

$$\frac{\partial c}{\partial \alpha_j} + \lambda \frac{\partial s}{\partial \alpha_j} = 0 \tag{13}$$

Equation (13) can be re-written (as derived in section II-C of reference 3) as:

$$\underset{\sim}{K}\left(\lambda, r(\omega_i)\right) \cdot (\vec{\Delta\alpha}_j) = \vec{T}\left(\lambda, \partial_j^o, r(\omega_i), R(\omega_i)\right) \tag{14}$$

where: $j = 1, \ldots n$ for the segments of the electron density profile.

and where:

$\underset{\sim}{K}$ = The coefficient matrix, and is a function of the trade-off parameter, "λ"; and also a function of the computed ionospheric reflection coefficients $r(\omega_i)$.

$\vec{T}$ = A vector which is a function of the above "λ", and r. It is also a function of the measured ionospheric reflection coefficients, $R(\omega_i)$, and a function of the previous electron density profile, α_j^o.

$(\vec{\Delta\alpha}_j)$ = A vector which represents the modifications to be added to each previous electron density profile during each step of the iteration scheme.

ω_i = The propagation frequency, i=1,...m.

The solutions to equation (14) are obtained by an iterative procedure. First, an initial profile, α_j^o (j=1,...n) is chosen and a value is assigned to the trade-off parameter, "λ". These values, along with functions of the full-wave calculations, $r(\omega_i)$, and data values, $R(\omega_i)$, are substituted and a solution $(\vec{\Delta\alpha}_j)$ is obtained as:

$$(\vec{\Delta\alpha}_j) = \left(\underset{\sim}{K}^{-1}\right) \cdot \vec{T} \tag{15}$$

These values of the $(\Delta\alpha_j)$'s are added to the (α_j^o)'s to get a new profile $(\vec{\alpha}_j)_{new}$. The sequence then continues with $(\vec{\alpha}_j)_{new}$ assuming the role of $(\vec{\alpha_j^o})$,

and a new value assigned to "λ". This leads to a new solution of $(\Delta\vec{\alpha}_j)$ and thus to a new electron density profile given by the relation:

$$\left(\vec{\alpha}_j\right)_{new} = \left(\vec{\alpha}_j^{o}\right)_{previous} + \left(\Delta\vec{\alpha}_j\right) \tag{16}$$

E. Assumptions Applied in the Inversion Procedure

In the inversion procedure several assumptions concerning the ionospheric parameters are made. In particular all parameters will be considered to be known except the electron density profile, which will be unknown, and is to be found from the data. The geomagnetic field parameters will be taken to be known, and the magnetic azimuth will be restricted to be either 90° or 270°. As is the usual case at VLF, ions are considered to have negligible effect on propagation and are omitted from the model. The collision frequency profile is taken to be known and is constrained to be of exponential form:

$$\nu = \nu_o \exp(-az) \tag{17}$$

Additional assumptions (i.e. constraints) on the form of the desired electron density profile are listed below and are illustrated in figure 1 where the electron density (N_e) is shown as a function of ionospheric height in terms of $\log_{10} N_e$.

(1) "Stop" constraints are introduced at both the top and bottom of the electron density profile. These correspond to limiting values of electron density which the profile may assume at these heights. At the top of the profile the "stop" defines the minimum value of electron density which the profile may assume. At the bottom, the "stop" defines the maximum value for that height.

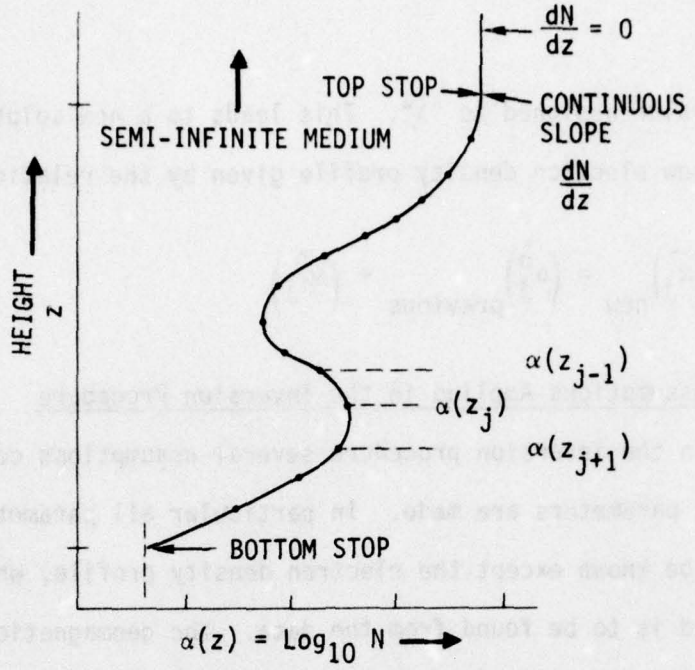


Figure 1. Segmented Electron Density Profile, $\alpha(z_j)$'s Showing Constraints.

(2) It is assumed that there are no electrons below the chosen lowest height of the profile. Above the chosen top height of the profile the electron density is assumed to be of constant value equal to the electron density at the chosen top height. This latter property is usually referred to as a semi-infinite medium.

(3) The profile is constrained to be reasonably smooth but otherwise consistent with the reflection coefficient data. Note that this implies that the profile approaches a slope of (dN/dz) = 0 near the top height which is just below the semi-infinite medium.

Inequalities are used as constraints on the end points of the $\alpha(z)$ profile. At the top of the profile the electron density is constrained to be at least as great as a chosen fixed value. At the bottom height of the

profile, the electron density is constrained to be no greater than a chosen fixed value. Thus:

$$\begin{aligned} \alpha(a) = \alpha(z_1) &\geq \alpha_{\text{Top Stop}} \\ \alpha(b) = \alpha(z_n) &\leq \alpha_{\text{Bot Stop}} \end{aligned} \tag{18}$$

In addition, in order to ensure a continuous slope, (dN_e/dz), leading upward to the semi-infinite medium, the slope at the top of the profile $(z = a = z_1)$ is constrained to approach that of the semi-infinite medium. In terms of the segmented $\alpha(z)$ curve, that is, the segmented electron density profile, this is implemented by adding the term:

$$(\alpha_2 - \alpha_1)^2 \, (\Delta z)^{-3} \tag{19,a}$$

to the summation representing the curvature function. That is, "c" is now defined as:

$$c = \left\{ (\alpha_2 - \alpha_1)^2 + \sum_{j=2}^{n-1} (\alpha_{j-1} - 2\alpha_j + \alpha_{j+1})^2 \right\} (\Delta z)^{-3} \tag{19,b}$$

Compare this result to equation (9).

With these sets of constraints, the equation to be solved is a modified form of equation (13). In particular the matrix:

$$\begin{array}{c} \\ j' \downarrow \end{array} \begin{array}{c} \overset{\quad j}{\downarrow} \\ \begin{pmatrix} (1+p_T) & -1 & 0 \ldots 0 \\ -1 & +1 & 0 \ldots 0 \\ 0 & 0 & 0 \ldots 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \ldots p_B \end{pmatrix} \end{array} \tag{20}$$

is added to the unmodified $\underset{\sim}{C}$ matrix of equation (11). That is, the original $\underset{\sim}{C}$ matrix terms are modified to be:

$$\left\{\begin{array}{ll} C_{11} = 1 + (1 + p_T) & = 2 + p_T \\ C_{12} = -2 + (-1) & = -3 \\ C_{21} = -2 + (-1) & = -3 \\ C_{22} = 5 + (+1) & = 6 \\ C_{nn} = 1 + p_B & = 1 + p_B \end{array}\right\} \tag{21}$$

The new matrix $\underset{\sim}{C}^{new}$ is defined as

$$\underset{\sim}{C}^{new} = \begin{pmatrix} (2+p_T) & -3 & 1 & 0 & . & . & . & . \\ -3 & 6 & -4 & 1 & 0 & & & \\ 1 & -4 & 6 & -4 & 1 & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & 1 & -4 & 6 & -4 & 1 & \\ & & & 1 & -4 & 5 & -2 & \\ & & & & 1 & -2 & (1+p_B) \end{pmatrix} \tag{22}$$

A vector, $\vec{V}$, is also defined to be:

$$\underset{j'}{\downarrow}\left(V_{j'_{new}}\right) = \underset{j'}{\downarrow}\begin{pmatrix} p_T \cdot \alpha_{Top\ Stop} \\ \\ p_B \cdot \alpha_{Bot\ Stop} \end{pmatrix} \tag{23}$$

and is a consequence of the added constraints.

The values of the inequalities of equation (18) are defined as follows:

(a) p_T {
- = 0, if in the solution for the $(\Delta\alpha_j)$'s of equation (15), the top end of the profile is inclined to move to the right of the stop.
- = a very large positive value if the top end of the profile is inclined to move to the left of the stop.

(b) p_B {
- = 0 if, in the solution for the $(\Delta\alpha_j)$'s of equation (15) the bottom end of the profile is inclined to move to the left of the stop.
- = a very large value if the bottom end of the profile is inclined to move to the right of the stop.

F. The Complex Reflection Coefficient Plane

Calculation performed using different electron density profiles will produce different values for the set of reflection coefficient ratios, $r(\omega_i)$.

The reflection coefficient ratio values are illustrated in figure 2 as plotted in the complex R (or r) plane. Here the frequency, ω, serves as the parametric variable. The data values, $R(\omega_i)$ are shown in comparison to two sets of $r(\omega_i)$ values. The curve $[r(\omega)]_1$ results from applying equation (4) and equation (5) to a given electron density profile. The curve $[r(\omega)]_2$ presents the results obtained from a second electron density profile. It is pointed out, that because of the constraints placed on the trial electron electron density profiles, not all forms of the $r(\omega)$ curve can be generated simply by varying the profile.

The problem to be solved with the inversion procedure is to find that electron density profile, which when used in the full-wave computation of reflection coefficients, will produce a match to the data, $R(\omega_i)$. Such a match is illustrated in figure 3. Because of error in the data

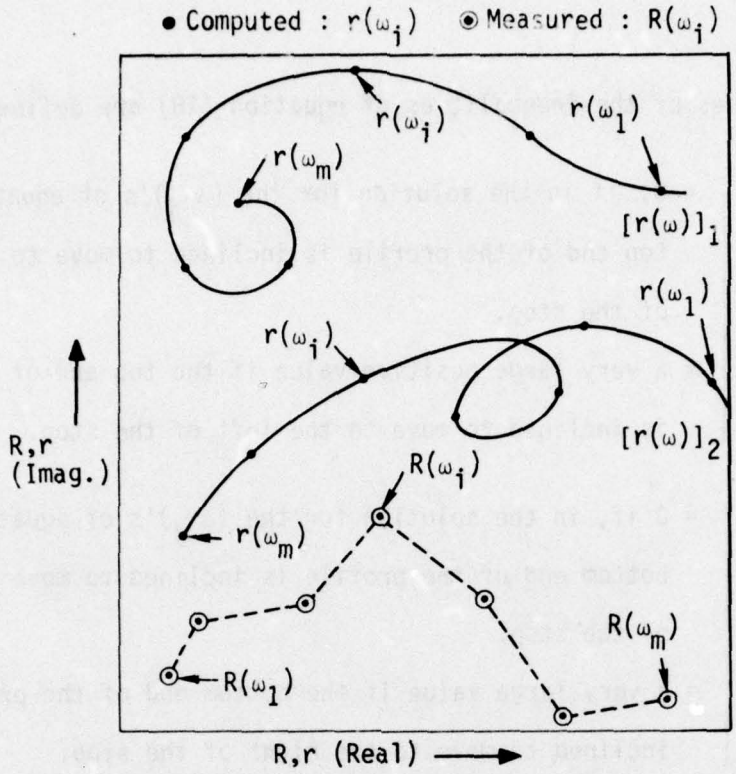


Figure 2. Data Values $R(\omega_i)$ and Computed Values $r(\omega_i)$ as Plotted in the Complex Plane.

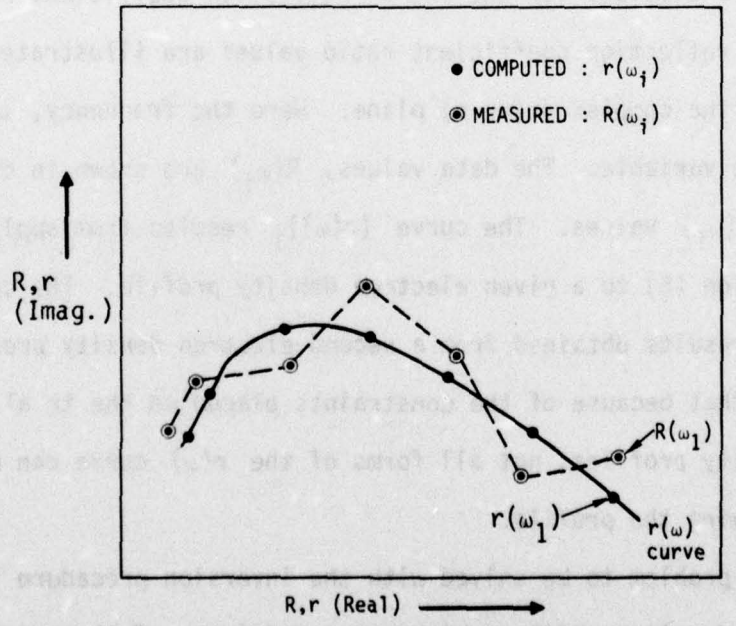


Figure 3. Matched Comparison of $R(\omega_i)$ Reflection Coefficient Data and $r(\omega)$ Computations.

and because of the constraints put on the ionospheric model, the data cannot be matched exactly.

The fact that an attempt is made to match the curves in real and imaginary parts of $R(\omega_i)$ and $r(\omega_i)$ implies that the spacing, in the R-r plane between frequency points (ω_i and ω_{i+1}, etc.) of $r(\omega_i)$ and $r(\omega_{i+1})$, etc., must also match the spacing between $R(\omega_i)$ and $R(\omega_{i+1})$, etc.

G. The Transformation Functions, $g(\omega_i)$ and $G(\omega_i)$

In order to make convergence of equation (14) more likely, the following function of the ionospheric reflection coefficients is introduced. (See the discussion on pp 28 in section II-C of reference 3.) Define:

$$g_1 = r(\omega_1)$$

$$g_i = \ln\left(\frac{dr}{d\omega}\right)_i \qquad (24)$$

$$i = 1, \ldots m$$

where

r_i is given by equation (5).

ω is the propagation frequency (kHz).

m is the total number of data frequencies.

Equation (24) may also be written as:

$$g_i = \ln\left|\left(\frac{dr}{d\omega}\right)_i\right| + j(\phi_g)_i \qquad (25)$$

$$i = 2, \ldots m$$

and

$$(\phi_g)_i = \left((\phi_g)_i \pm 2n_i\,\pi\right) \tag{26}$$

where (ϕ_g) is the principal part of the phase term and lies in the interval $-\pi < \phi_i \leq \pi$.

The value of n_i, in the phase term, is chosen for each frequency so as to ensure that the phase, ϕ_i, is continuous along the $r(\omega)$ curve.

The phase angle $(\phi_g)_i$ may also be written as:

$$(\phi_g)_i = \text{ARCTAN}\left(\frac{\text{Imag. } (dr/d\omega)_i}{\text{Real } (dr/d\omega)_i}\right) \tag{27}$$

The definition of the transformation function, as given by equation (25), allows for a complete definition of the $r(\omega)$ curve in terms of spacing (i.e., $|dr/d\omega|$), phase (i.e., ϕ) and the location of one point on the $r(\omega)$ curve, where for example the one frequency point is chosen to be the lowest frequency, ω_1. An $r(\omega)$ curve illustrating the "g" transformation is given in figure 4.

The transformation function for the data values, $R(\omega_i)$, is also introduced as:

$$\left.\begin{aligned} G_1 &= R(\omega_1) \\ G_i &= \ln\left(\frac{dR}{d\omega}\right)_i \\ &\text{or} \\ G_i &= \ln\left|\left(\frac{dR}{d\omega}\right)_i\right| + j(\phi_G)_i \\ i &= 2, \ldots m \end{aligned}\right\} \tag{28}$$

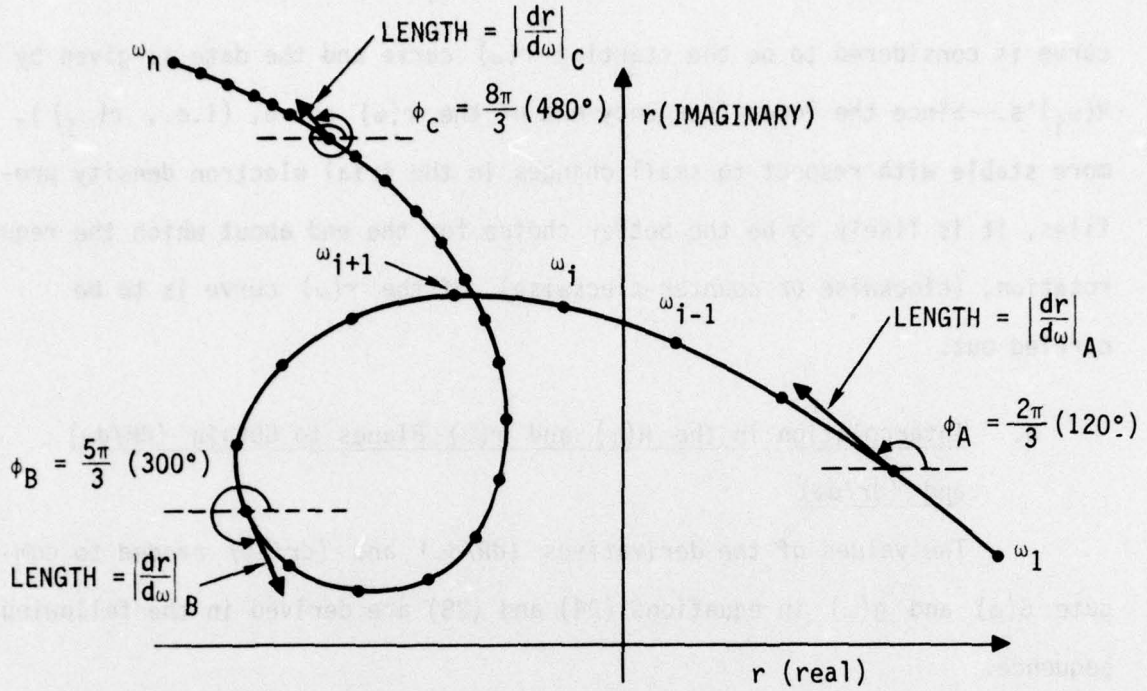


Figure 4. The $r(\omega)$ Curve Illustrating the Transformation Function, $g(\omega_i) = \ln (dr/d\omega)_i + j\phi_i$ (i.e., Spacing and Phase).

The phase, $(\phi_G)_i$ is written as:

$$(\phi_G)_i = \left((\Phi_G)_i \pm 2 (n_G)_i \pi\right) \tag{29}$$

and

$$(\phi_G)_i = \text{ARCTAN}\left(\frac{\text{Imag. } (dR/d\omega)_i}{\text{Real } (dR/d\omega)_i}\right) \tag{30}$$

To obtain a match between an $r(\omega)$ curve and the data curve $R(\omega)$, as exemplified in figure 3, the required $r(\omega)$ curve may need to be completely reversed from the $r(\omega)$ curve obtained from the starting solution of the inversion scheme. This is the situation portrayed in figure 2 where the $[r(\omega)]_2$

curve is considered to be the starting $r(\omega)$ curve and the data is given by the $R(\omega_i)$'s. Since the lower frequency end of the $r(\omega)$ curve, (i.e., $r(\omega_1)$), is more stable with respect to small changes in the trial electron density profiles, it is likely to be the better choice for the end about which the required rotation, (clockwise or counter-clockwise), of the $r(\omega)$ curve is to be carried out.

H. Interpolation in the $R(\omega)$ and $r(\omega)$ Planes to Obtain $(dR/d\omega)$ and $(dr/d\omega)$

The values of the derivatives $(dR/d\omega)$ and $(dr/d\omega)$ needed to compute $G(\omega)$ and $g(\omega)$ in equations (24) and (28) are derived in the following sequence.

The set of transmitted frequencies, ω_i, $(i = 1, \ldots m)$ is assumed to be a subset of possible frequencies, $\bar{\omega}_k$, $(k = 1, \ldots K)$. K is larger than m. A reflection coefficient ratio (similar to $r(\omega_i)$) will be denoted for the frequency set, $\bar{\omega}_k$, by $\rho(\bar{\omega}_k)$. The defining equation is:

$$\underset{\downarrow}{k}\left(\rho(\omega_k)\right) = \underset{\downarrow}{k}\left|b_{k,i}\right| \cdot \overset{i\rightarrow}{\underset{\downarrow}{i}}\left(R(\omega_i)\right) \tag{31}$$

Where the $\rho(\bar{\omega}_k)$'s are linear combinations of the $R(\omega_i)$'s with coefficients $(b_{k,i})$. The $\rho(\bar{\omega}_k)$ values are obtained by interpolation of the $R(\omega_i)$ values and equation (31) is the interpolation equation. The $(b_{k,i})$ are defined in section II-C of reference 3. The $(b_{k,i})$ are not functions of the numerical values of the $R(\omega_i)$ but depend only on the spacing of the ω_i frequencies and on the number of frequencies m. At any frequency value where ω_i is equal to an $\bar{\omega}_k$, the value of $\rho(\bar{\omega}_k)$ will be equal to the value of $R(\omega_i)$.

Figure 5 shows the R-plane representation of the interpolated values, $\rho(\bar{\omega}_k)$, and the data value, $R(\omega_i)$.

The derivative at any one interpolation point, $\bar{\omega}_k$, is defined in terms of the change in value of $\rho(\bar{\omega})$ at the interpolation points preceding and following that point. That is:

$$\left(\frac{d\rho(\omega)}{d\omega}\right)_k = \frac{\left(\rho(\bar{\omega}_{k+1}) - \rho(\omega_{k-1})\right)}{2 \cdot \Delta\omega} \tag{32}$$

where $\Delta\omega$ is the interpolation increment in frequency.

Using equation (31) equation (32) is also written:

$$\left(\frac{d\rho(\omega)}{d\omega}\right)_k = (b_{k+1,i} - b_{k-1,i})\ R(\omega_i)/2\Delta\omega \tag{33}$$

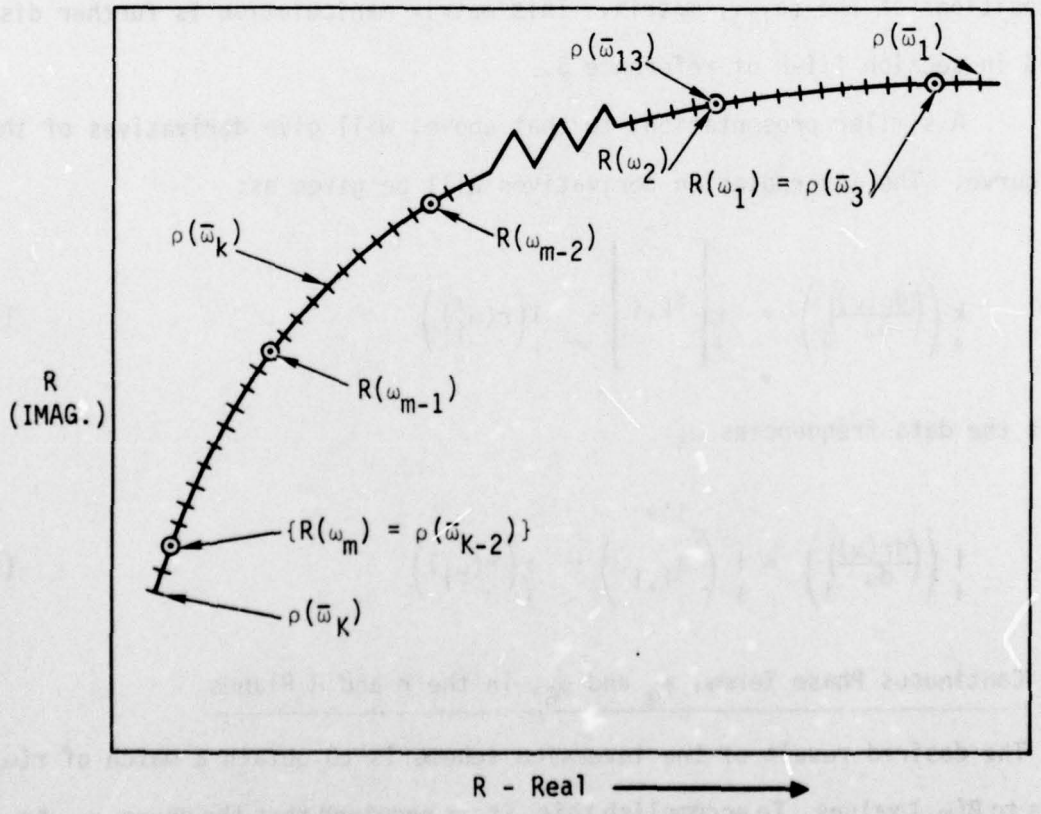


Figure 5. R-Plane Representation of Interpolated Values of $\rho(\omega_k)$ and Data Values $R(\omega_i)$.

and in matrix notation

$$\left.\begin{array}{l} \underset{k}{\downarrow}\left(\left(\frac{d\rho(\omega)}{d\omega}\right)_k\right) = \underset{k}{\downarrow}\left[\overset{i\rightarrow}{\beta_{k,i}}\right] \cdot \underset{i}{\downarrow}\left(R(\omega_i)\right) \\ \text{where} \quad \beta_{k,i} = (b_{k+1,i} - b_{k-1,i}) \end{array}\right\} \tag{34}$$

At the data frequencies, ω_i, equation (34) simplifies to:

$$\underset{i}{\downarrow}\left(\left(\frac{dR(\omega)}{d\omega}\right)_i\right) = \underset{i}{\downarrow}\left(\overset{i'\rightarrow}{a_{ii'}}\right) \cdot \underset{i'}{\downarrow}\left(R(\omega_i)\right) \tag{35}$$

where the $a_{i,i'}$ matrix is obtained from the $\beta_{k,i}$ matrix of equation (34) by placing only those rows of the $(\beta_{k,i})$ matrix which correspond to ω_i values into row positions of the $(a_{ii'})$ matrix. This matrix manipulation is further discussed in section III-H of reference 3.

A similar presentation, to that above, will give derivatives of the $r(\omega)$ curve. The interpolation derivatives will be given as:

$$\underset{k}{\downarrow}\left(\left(\frac{d\rho(\omega)}{d\omega}\right)_k\right) = \underset{k}{\downarrow}\left[\overset{i\rightarrow}{\beta_{k,i}}\right] \cdot \underset{i}{\downarrow}\left(r(\omega_i)\right) \tag{36}$$

and at the data frequencies ω_i:

$$\underset{i}{\downarrow}\left(\left(\frac{dr(\omega)}{d\omega}\right)_i\right) = \underset{i}{\downarrow}\left(\overset{i'\rightarrow}{a_{i,i'}}\right) \cdot \underset{i}{\downarrow}\left(r(\omega_i)\right) \tag{37}$$

I. Continuous Phase Terms, ϕ_g and ϕ_G, in the r and R Planes

The desired result of the inversion scheme is to obtain a match of $r(\omega_i)$ values to $R(\omega_i)$ values. To accomplish this, it is required that the phase, ϕ_g, be continuous along the $r(\omega)$ curve. The same is true for the phase ϕ_G along the $R(\omega)$ curve.

From equation (25) the $g(\omega)$ transformation variable may be written in terms of the interpolated variables, $\rho(\bar{\omega}_k)$, as:

$$\bar{g}_k = \ln\left|\frac{d\rho}{d\omega}_k\right| + j\underbrace{\left((\phi_g)_k \pm 2\,(n_g)_k\,\pi\right)}_{(\phi_g)_k} \tag{38}$$

and from equation (28) the expression for $G(\omega)$

$$\bar{G}_k + \ln\left|\frac{d\rho}{d\omega}_k\right| + j\underbrace{\left((\phi_G)_k \pm 2(\bar{n}_G)_k\,\pi\right)}_{(\phi_G)_k} \tag{39}$$

To satisfy the matching requirement between the $r(\omega)$ and $R(\omega)$ curves the values of $(\bar{n}_g)_K$ and $(\bar{n}_G)_k$ must be chosen for each frequency, $\bar{\omega}_k$, so that the phases ϕ_g and ϕ_G are continuous along the $r(\omega)$ and $R(\omega)$ curves. To insure continuous changes in phase along the curves the differences: $|\phi_k - \phi_{k-1}|$ and $|\phi_{k+1} - \phi_k|$ must be small.

The smoothing requirement for the continuous phase along the $r(\omega)$ curve leaves the value of $(\bar{n}_g)_k$ arbitrary for one frequency, e.g. the lowest frequency, ω_1, which is $\bar{\omega}_3$. The value of $(\bar{n}_g)_{k=3}$ is then chosen so as to cause the $r(\omega)$ curve to rotate in the desired sense, (i.e. clockwise or counter clockwise) to obtain a match to the $R(\omega)$ data curve. The values of the phases $(\phi_G)_k$ and $(\phi_g)_k$ at the point k=3 will be identified as ϕ_D (for data phase) and ϕ_C (for computed phase) respectively.

Figure 6 illustrates some possible cases for the phase relations between the data curve, $R(\omega)$, and two examples of computed curves, $[r(\omega)]_1$ and $[r(\omega)]_2$. The phase values are identified as the direction assumed by a vector

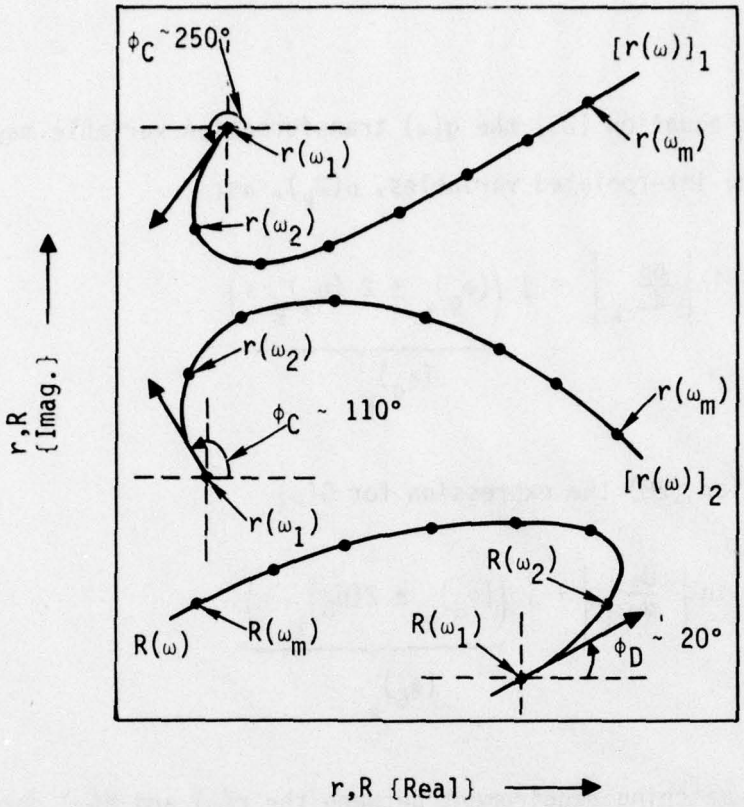
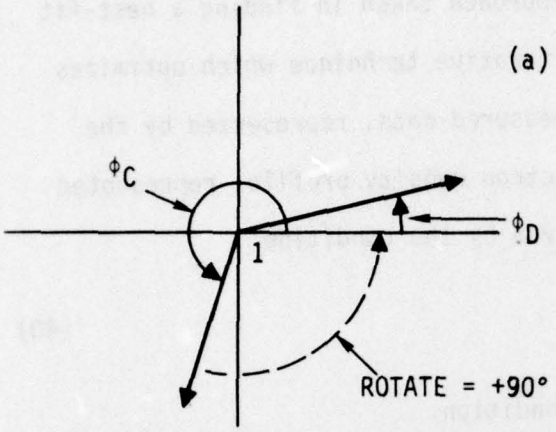
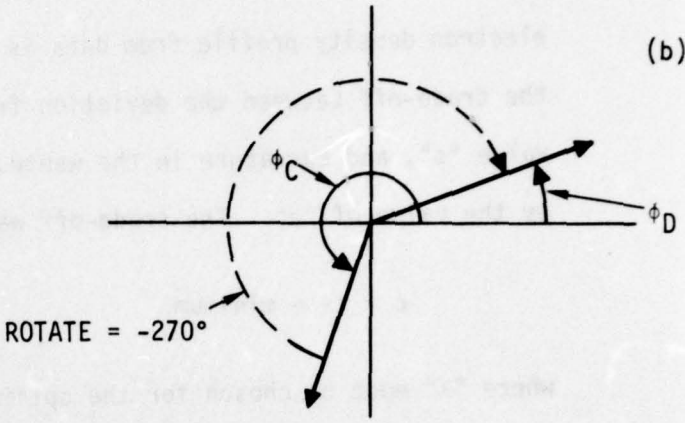


Figure 6. Examples of ϕ_D for Data Values, $R(\omega)$, and ϕ_C for Computed Values, $r(\omega)$.

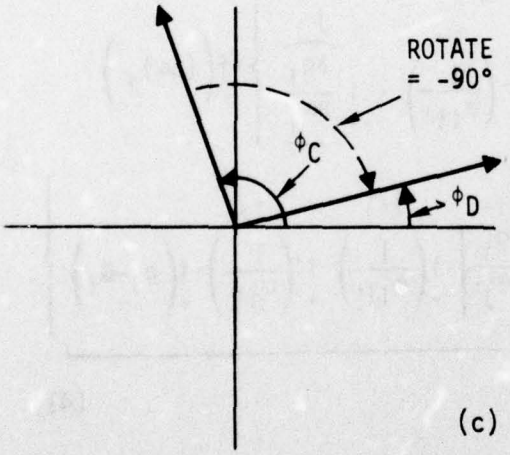
originating at the lowest frequency point, $r(\omega_1)$ (or $R(\omega_1)$) directed tangent to the $r(\omega)$ (or $R(\omega)$) curve as the curve extends toward the point $r(\omega_2)$ (or $R(\omega_2)$. There are two ways that the $[r(\omega)]_1$ (or [(the $r(\omega)]_2$) curve can be transformed to match the $R(\omega)$ curve. These are a clockwise rotation or a counter-clockwise rotation of the $r(\omega)$ curve about the point $r(\omega_1)$. This rotation is accomplished in the INVERT computer program by the variable ROTATE. A positive value assigned to ROTATE results in a counter-clockwise rotation while a negative value gives a clockwise rotation. Examples of the choices of values for ROTATE are presented in figure 7. The procedure to be used in executing the program is to try one direction of rotation and, if it does not prove successful, try the other direction.



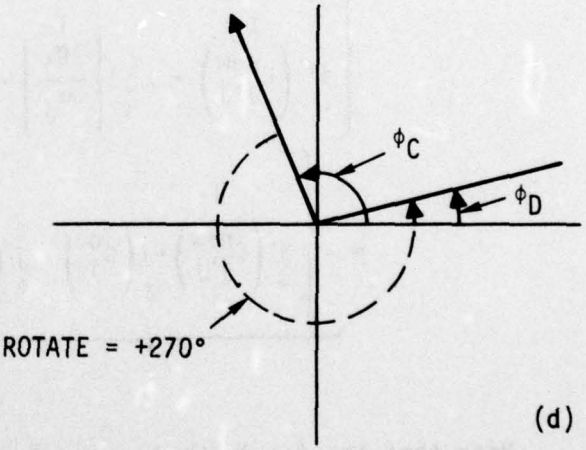
Counterclockwise rotation for the curve $[r(\omega)]_1$ toward the curve $[R(\omega)]$.



Clockwise rotation for the curve $[r(\omega)]_1$ toward the curve $[R(\omega)]$.



Clockwise rotation for the curve $[r(\omega)]_2$ toward the curve $R(\omega)$.



Counterclockwise rotation for the curve $[r(\omega)]_2$ toward the curve $R(\omega)$.

Figure 7. Examples of the Choices of Values for the ROTATE Variable to Cause the Curve $r(\omega)$ to Rotate Toward the $R(\omega)$ Curve.

J. The Inversion Equation

As stated in section II,D, the approach taken in finding a best-fit electron density profile from data is an iterative technique which optimizes the trade-off between the deviation from measured data, represented by the value "s", and curvature in the wanted electron density profile, represented by the value of "c". The trade-off was given by the condition:

$$c + \lambda s \rightarrow \text{minimum} \tag{40}$$

where "λ" must be chosen for the optimum condition.

The actual equation to be solved in order to obtain the desired electron density profile is derived in section II-C of reference 3 as:

$$\overbrace{\left\{ \underset{\downarrow}{j'} \left(C^{new}_{j'j} \right)^{\underset{\rightarrow}{j}} + \lambda \underset{\downarrow}{j'} \left[\frac{\partial g_i}{\partial \alpha_j} \right]^{\underset{\rightarrow}{i}} \cdot \underset{\downarrow}{i} \left(\frac{1}{\sigma_{ii'}} \right)^{\underset{\rightarrow}{i'}} \cdot \underset{\downarrow}{i'} \left(\frac{1}{\sigma_{ii'}} \right)^{\underset{\rightarrow}{i}} \cdot \underset{\downarrow}{i} \overbrace{\frac{\partial g_i}{\partial \alpha_j}}^{\underset{\rightarrow}{j}} \right\}}^{\text{coefficient matrix, } \underset{\sim}{K}} \cdot \underset{\downarrow}{j} \left((\Delta\alpha)_j \right)$$

$$= \underbrace{- \left\{ \underset{\downarrow}{j'} \left(C^{new}_{j'j} \right)^{\underset{\rightarrow}{j}} \cdot \underset{\downarrow}{j} \left(\partial^o_j \right) - \underset{\downarrow}{j} \left(V_j \right) + \lambda \underset{\downarrow}{j'} \left[\frac{\partial g_i}{\partial \alpha_{j'}} \right]^{\underset{\rightarrow}{i}} \cdot \underset{\downarrow}{i} \left(\frac{1}{\sigma_{ii'}} \right)^{\underset{\rightarrow}{i'}} \underset{\downarrow}{i'} \left(\frac{1}{\sigma_{ii'}} \right)^{\underset{\rightarrow}{i}} \cdot \underset{\downarrow}{i} \left(g_i - G_i \right) \right\}}_{\text{T-vector}} \tag{41}$$

Note that the $(\Delta\alpha_j)$ matrix and the (α^o_j) matrix can also be denoted as the vectors $(\vec{\Delta\alpha}_j)$ and $(\vec{\alpha}^o_j)$ respectively. Eq. (41) may be written in the form of equation (14) as

$$\underset{\sim}{K} \left(\lambda, g(\omega_i) \right) \cdot (\vec{\Delta\alpha}_j) = \vec{T} \left(\lambda, \alpha^o_j, g(\omega_i), G(\omega_i) \right) \tag{42}$$

with solutions

$$(\vec{\Delta\alpha}_j) = (\underset{\sim}{K}^{-1}) \cdot \vec{T} \tag{43}$$

Note that the coefficient matrix, $\underset{\sim}{K}$, must be non-singular for a solution, $(\vec{\Delta\alpha}_j)$, to be obtained.

The $C^{new}_{jj'}$ matrix of equation (41) is given by equation (22), the $\vec{V}$ vector is given by equation (23), and the g_i and G_i terms are given by equations (24) and (28) respectively. The uncertainty matrix $(1/\sigma_{ii'})$ is derived in appendix D of reference 3, where the resulting equations are:

$$\begin{matrix} i' \downarrow \\ \end{matrix} \left(\frac{1}{\sigma_{i'i}} \right)_{new} = \begin{matrix} i' \downarrow \\ \end{matrix} \left(\left(\frac{\partial g_{i'}}{\partial r_i} \right)^{-1}_{r=k} \right) \cdot \begin{matrix} i \downarrow \\ \end{matrix} \begin{pmatrix} & i \rightarrow & \\ & \frac{1}{\sigma_{ii}} & 0 \\ 0 & & \end{pmatrix} \tag{44}$$

$$\begin{matrix} i \downarrow \\ \end{matrix} \left(\frac{1}{\sigma_{ii'}} \right)_{new} = \begin{matrix} i \downarrow \\ \end{matrix} \begin{pmatrix} & i \rightarrow & \\ & \frac{1}{\sigma_{i'i}} & 0 \\ 0 & & \end{pmatrix} \cdot \begin{matrix} i' \downarrow \\ \end{matrix} \left(\left(\frac{\partial g_{i'}}{\partial r_i} \right)^{-1}_{r=R} \right) \tag{45}$$

where the matrix:

$$\begin{matrix} i \downarrow \\ \end{matrix} \begin{pmatrix} & i \rightarrow & \\ & \frac{1}{\sigma_{ii}} & 0 \\ 0 & & \end{pmatrix} \tag{46}$$

is a diagonal matrix with real constant elements and is composed of the expected uncertainties as assigned to the data values, $R(\omega_i)$. The expression "r=R" implies that, in the derivatives $(\partial g/\partial r)$, everywhere an r_i occurs replace it with R_i (i.e. a data value).

The derivative terms $(\partial g/\partial r)$ are derived in appendix E of reference 3 as:

$$\left(\frac{\partial g_{i'}}{\partial r_i}\right) = \left\{ \frac{a_{i'\,i}}{\sum_{i''=1}^{m} a_{i'\,i''}\, r_{i''}} \right\} \tag{47}$$

$i' = 2,\ldots m$

$i = 1,\ldots m$

and the $a_{ii'}$ factors are defined in equation (35).

The derivative terms $(\partial g_i/\partial \alpha_j)$ are derived in section III-I of reference 3 as

$$\left(\frac{\partial g_i}{\partial \alpha_j}\right) = \frac{\left(\sum_{i'}^{n} (a_{ii'}) \frac{\partial r_i}{\partial \alpha_j} \right)}{\left(\sum_{i'}^{n} a_{ii'}\, r_i \right)} \tag{48}$$

where the derivative $(\partial r_i/\partial \alpha_j)$ is given in section IV-E of reference 3 as:

$$\frac{\partial r_i}{\partial \alpha_j} = 2\,J \left\{ \frac{{}_{\perp}R_{\parallel} \frac{\partial {}_{\perp}R_{\perp}}{\partial \alpha_j} - {}_{\perp}R_{\perp} \frac{\partial {}_{\perp}R_{\parallel}}{\partial \alpha_j}}{(-\,{}_{\perp}R_{\perp} - J\;{}_{\perp}R_{\parallel})^2} \right\} \tag{49}$$

$J = \sqrt{-1}$

III. SEQUENCE OF PROGRAM EXECUTION

A more explicit description of the inversion technique used in the INVERT computer program is as follows.

First, the initial electron density profile is defined to have a constant value, $\alpha(z)$, for all heights from the bottom height through the top height. The electron density of this profile is computed for all z_j as:

$$\alpha(z)_j = \frac{\alpha(\text{Top stop}) - \alpha(\text{Bottom stop})}{2} \tag{50}$$

This set of $\alpha(z)_j$'s is defined as (α_j^o) initial. Note that the height increments chosen for the profile must satisfy the following relationship:

$$\frac{z(\text{Top stop}) - z(\text{Bottom stop})}{\text{DELTAH}} = n \cdot 2 \tag{51}$$

where n is an integer and DELTAH is an initial value input to the computer program.

In the matrix equation (41), the first solution desired is for "λ" = $\lambda_1 = 0$. This requires:

$$\underset{\downarrow}{j}\left(C_{jj'}^{new}\right) \cdot \underset{\downarrow}{j}\left(\Delta\alpha_j\right) + \underset{\downarrow}{j'}\left(C_{jj'}^{new}\right) \cdot \underset{\downarrow}{j}\left(\alpha_j^o\right)_{Initial} = \underset{\downarrow}{j}\left(V_j\right) \tag{52}$$

where

$$(\Delta\vec{\alpha}_j) = (\vec{\alpha}_j)_{new} - (\vec{\alpha_j^o})_{initial} \tag{53}$$

and

$$j = 1,\ldots n$$

Substitution of equation (53) into equation (52) gives the solution:

$$\underset{j\downarrow}{\left((\alpha_j)_{new}\right)} = \underset{j'\downarrow}{\left(C^{new}_{jj'}\right)} \cdot \underset{j\downarrow}{\left(V_j\right)} \qquad (54)$$

This implies that a solution for $(\vec{\alpha}_j)_{new}$ is possible as long as the $\underset{\sim}{C}^{new}$ matrix is non-singular. For the next step in the inversion, the above $(\vec{\alpha}_j)_{new}$ becomes $(\vec{\alpha}_j)_1$, which is identified as the profile, α^1.

A full-wave computation, using the just defined set of profile values, $(\vec{\alpha}_j)_1$, as the input profile will give the set of reflection coefficients, $r(\omega_i)_1$ (or really $g(\omega_i)_1$ and the derivatives $(\partial r_i/\partial \alpha_j)_1$ (or $(\partial g_i/\partial \alpha_j)_1$) needed to proceed with the inversion scheme.

The next step in the procedure is to substitute the values of $(\vec{\alpha}_j)_1$ for (α^o_j) along with the values $g(\omega_i)_1$, $(\partial g_i/\partial \alpha_j)_1$, and "$\lambda$" (i.e. $\lambda=\lambda_2=\lambda_1=0$) all into equation (41) and solve the equation for $(\Delta\vec{\alpha}_j)$. The $(\Delta\vec{\alpha}_j)$ solution will be identified as $(\Delta\vec{\alpha}_j)_2$. This yields what will be called a "settle down" profile (i.e. $(\vec{\alpha}_j)_2 = (\vec{\alpha}_j)_1 + (\Delta\vec{\alpha}_j)_2$). The use of the "settle down" profile in the inversion sequence will be discussed later.

Note that the variables, λ_{odd}, will be used to identify the increasing sequence of "λ" values used in the inversion procedure. Also, the terms, $\Delta\lambda_{odd}$, will be used to increment the λ_{odd} values. The variables λ_{even} will be used to identify the "settle down" steps in the inversion procedure. A relationship is that λ_{even} is equal to the previous λ_{odd} (i.e. $\lambda_2 = \lambda_1$, $\lambda_4 = \lambda_3$ etc.). This implies that $\lambda_2 = \lambda_1 = 0$. The term $\Delta\lambda_{even}$ will not be identified or used in the procedure. The term $\Delta\lambda_1$ is an initial value assigned to the computer program as the variable DELTAL.

The inversion process is illustrated in figure 8. Here the curve shows the variation of the profile, $\alpha(z)$ with the variable, "λ", as given by the minimization condition of equation (12). The objective is to follow the curve by increasing the value of "λ" with the restriction being to stay as close to the curve as possible. The process is halted when a predetermined criteria is met. This criteria will be discussed later. The figure shows the basic steps involved in the iterative sequence to obtain the desired final profile.

In figure 8, the point "a" is identified as the profile, α^1 (which is the set of (α_j) values given by the previously defined vector $(\vec{\alpha}_j)_1$. To get from point "a" to point "b", in figure 8, a new value is assigned to the variable, "λ". This is given by $\lambda_{new} = \lambda_3 = \lambda_1 + \Delta\lambda_3$. The value $\Delta\lambda_3$ is related to $\Delta\lambda_1$. At point "b", the values of the variables $g(\omega_i)_1$, $(\partial g_i/\partial \alpha_j)_1$, λ_3 and the vector $(\vec{\alpha}_j)_1$ are all substituted into equation (41). The $(\alpha_j)_1$ values are

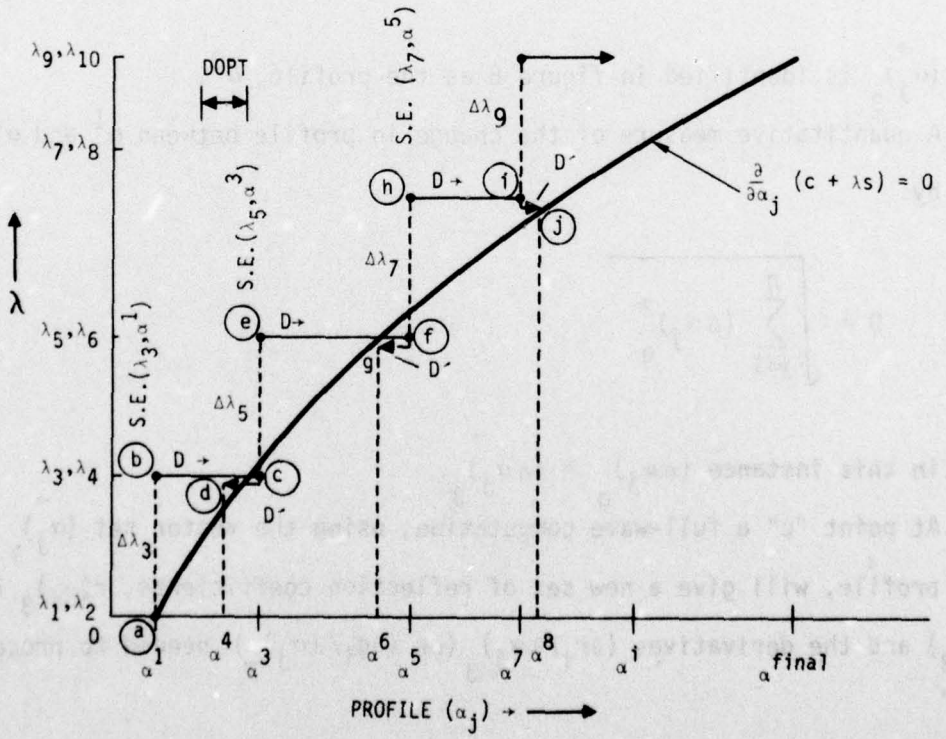


Figure 8. Iterative Sequence to Obtain Desired Profile, $(\alpha)^{final}$.

substituted for the factor $(\alpha_j^{\vec{0}})$. The only unknowns in the equations are the $(\Delta\alpha_j)$'s. Equation (41) is then solved for the vector, $(\vec{\Delta\alpha_j}) \equiv (\vec{\Delta\alpha_j})_3$. For a solution to be possible, the coefficient matrix, K of equation (41) must be non-singular.

Note that, throughout the remainder of the discussion of the inversion procedure, a solution to equation (41) will be denoted as SOLEQS (or S.E.). This is the name of the subroutine used in the INVERT computer program to solve for the $(\Delta\alpha_j)$'s.

A new electron density profile is given by the vector equation:

$$(\vec{\alpha_j})_{new} = (\vec{\alpha_j})_{previous} + (\vec{\Delta\alpha_j})_{new} \tag{55}$$

This gives, at the point "c" in figure 8, the new profile $(\vec{\alpha_j})_3$. That is

$$(\vec{\alpha_j})_3 = (\vec{\alpha_j})_1 + (\vec{\Delta\alpha_j})_3 \tag{56}$$

where $(\vec{\alpha_j})_3$ is identified in figure 8 as the profile, α^3.

A quantitative measure of the change in profile between α^1 and α^3 is given by:

$$D = \sqrt{\sum_{j=1}^{n} (\Delta\alpha_j)_q^2} \tag{57}$$

where in this instance $(\Delta\alpha_j)_q \equiv (\Delta\alpha_j)_3$.

At point "c" a full-wave computation, using the vector set $(\vec{\alpha_j})_3$ as the input profile, will give a new set of reflection coefficients, $r(\omega_i)_3$ (or $g(\omega_i)_3$) and the derivatives $(\partial r_i/\partial\alpha_j)_3$ (or $(\partial g_i/\partial\alpha_j)_3$) needed to proceed

with the inversion technique. If the profile α^3 lies close enough to the minimization curve of figure 8, then, the choice of value for λ_3 is acceptable and the inversion sequence may continue.

The requirement for acceptance of the α^3 profile is that the following equation must be true. That is:

$$\left| \bar{g}(\omega_i)_3 - \left\{ \bar{g}(\omega_i)_1 + \sum_{j=1}^{2n} \left(\frac{\partial \bar{g}_i}{\partial \alpha_j} \right)_1 (\Delta \alpha_j)_3 \right\} \right| \approx 0 \qquad (58)$$

$$i = 1, \ldots m$$

Here the $\bar{g}$'s are real functions of the variables g_i(real) and g_i(imaginary), (thus the reason for the 2n instead of n on the summation). Also, the $\bar{g}$'s contain the uncertainty matrices, $(1/\sigma)$, of equations (44) and (45).

A more thorough explanation of the meaning of expression (58) is the following. First, let a variable $\bar{g}'(\omega_i)$ be defined as:

$$\bar{g}'(\omega_i)_3 = \bar{g}(\omega_i)_1 + \sum_{j=1}^{2m} \left\{ \left(\frac{\partial \bar{g}_i}{\partial \alpha_j} \right)_1 (\Delta \alpha_j)_3 \right\} \qquad (59)$$

This is a linear first order relationship between the reflection coefficient function $\bar{g}(\omega_i)_1$ for profile α^1 and the reflection coefficient function $\bar{g}'(\omega_i)_3$ for profile α^3[1]. Now, the values computed for the $\bar{g}(\omega_i)_3$ variables using the full-wave integration calculation will not necessarily be linearly related to the $\bar{g}(\omega_i)_1$ results. They will, in general, be described by second and higher order terms being added to the first order linear term, as are presented in

equation (59). The differences between the linear variables $\bar{g}'(\omega_i)$ and the full-wave variables $\bar{g}(\omega_i)$ are thus related to the magnitude of the higher order terms. That is:

$$\left\{ \begin{array}{c} \bar{g}(\omega_i)_3 \quad - \quad \bar{g}'(\omega_i)_3 \\ \\ \text{full-wave} \qquad \text{equation (59)} \end{array} \right\} = \begin{array}{c} \text{2nd and higher order} \\ \text{terms are } (\Delta\alpha_j)_3 \end{array} \tag{60}$$

If the magnitudes of these higher order terms are unimportant, $\bar{g}(\omega_i)_3$ will be very close in value to $\bar{g}'(\omega_i)_3$ and the choice of λ_3 is probably not too large. On the other hand, if the difference between $\bar{g}(\omega_i)_3$ and $\bar{g}'(\omega_i)_3$ is large (see equation (58)) the choice of λ_3 is too large and will have to be reduced if the resulting α^3 profile is to remain close enough to the minimization curve.

It is important to point out at this time that small changes in the profile (i.e., $\Delta\alpha_j$) can result in very large changes in the reflection coefficients, as computed by the full-wave integration method, because of the addition of the higher order terms mentioned above.

To determine whether the requirement of equation (58) is met sufficiently, the following criteria is introduced and identified as a "SETTLE-DOWN" iteration. To carry out the "SETTLE-DOWN" procedure, the SOLEQS routine is executed, at the point "c" of figure 8, using the values $g(\omega_i)_3$, $(\partial g_i/\partial\alpha_j)_3$, $(\vec{\alpha}_j)_3$ and $\lambda = \lambda_4 = \lambda_3$. The solution is the vector $(\vec{\Delta\alpha}_j)_4$ which may be denoted as $(\vec{\Delta\alpha}_j)_{SD}$. The subscript, SD, stands for "SETTLE-DOWN."

The $(\vec{\Delta\alpha_j})_{SD}$ may be thought of as those modifications which when added to the previous profile, α^3, will give the "SETTLE-DOWN" profile, $(\vec{\alpha_j})_4 = \alpha^4$. That is

$$(\vec{\alpha_j})_4 = (\vec{\alpha_j})_3 + (\vec{\Delta\alpha_j})_{SD} \tag{60}$$

Note the location of the profile, α^4, in figure 8. This is identified as point "d". The values obtained for the $(\vec{\Delta\alpha_j})_{SD}$ are used to determine a measure of the nonlinearity of the $g(\omega_i)$ reflection coefficients with respect to the changes in the profile, (α_j).

Note here that all factors λ_{odd}, $\Delta\lambda_{odd}$ $(\vec{\alpha_j})_{odd}$, $(\vec{\Delta\alpha_j})_{odd}$ and α^{odd} refer to the full-wave related variables. The factors λ_{even}, $(\vec{\alpha_j})_{even}$, $(\vec{\Delta\alpha_j})_{even}$ and α^{even} refer to "SETTLE-DOWN" variables.

The purpose of the "SETTLE-DOWN" calculation is to define a quantitative measure of how acceptable the α^3 profile is for satisfying the minimization curve. Helpful functions for determining this requirement are:

$$(\bar{g}_i)_{SD} = \bar{g}(\omega_i)_{FW} + \sum_{j=1}^{2n} \left(\frac{\partial g_j}{\partial \alpha_j}\right)_{FW} (\Delta\alpha_j)_{SD} \tag{61}$$

$$i = 1, \ldots m$$

where the relation between $\bar{g}$ and g is the same as defined for equation (58). The subscript, FW, stands for the previous full-wave solution for the g's.

For the case of the profile α^3 equation (61) is:

$$(\bar{g}_i)_{SD} = \bar{g}(\omega_i)_3 + \sum_{j=1}^{2n} \left(\frac{\partial \bar{g}_i}{\partial \alpha_j}\right)_3 (\Delta\alpha_j)_4 \tag{62}$$

Equation (61) states that $\bar{g}_{SD}$ is computed as a linear function of the $\bar{g}_i$'s. A second function determined from the "SETTLE-DOWN" results is:

$$S' = \sum_{i=1}^{m} \left((\bar{g}_i)_{SD} - G_i\right)^2 \tag{63}$$

where G_i is a data value as given by equation (28). The expression S' will be used later in determining where the interactive procedure is to be stepped.

The variable used to give a quantitive measure to the "SETTLE-DOWN" criteria is given as

$$\text{D PRIME} = \sqrt{\sum_{j=1}^{n} (\Delta\alpha_j)^2_{SD}} \tag{64}$$

If the optimum magnitude of the "SETTLE-DOWN" is not exceeded then the choice of λ^3 was not too large and the profile α^3 is close enough for the inversion sequence to continue. The variable D PRIME is indicated in figure 8 by D'.

In order to continue at an optimum rate of progress (i.e., incrementing "λ" so as to move along the minimization curve), or to back up (i.e., decreasing the value of "λ"), the following procedure is used by the INVERT program.

A "threshold" term is defined by the equation

$$\text{RATIO} = \frac{\text{D PRIME}}{\text{D OPT}} \tag{65}$$

The numerical results of equation (65) is compared to the set of iteration rules presented in table I. These rules state how "λ" is to be incremented (i.e., $\lambda_{next} = \lambda_{previous} + \Delta\lambda$) for the next execution of the SOLEQS routine in order for the inversion process to continue.

Assuming that the value of RATIO gives a favorable result as, determined by the rules of table I, an increment $\Delta\lambda_5$ is added to λ_3 to get to point "e" of figure 8 and the inversion procedure continues.

From the above discussion it is seen that the inversion scheme proceeds in a "boot strap" fashion of obtaining solutions $(\vec{\Delta\alpha_j})$ to equation (41) for a particular value of "λ", computing a new profile $((\vec{\alpha_j})_{new} = (\vec{\alpha_j})_{old} + (\vec{\Delta\alpha_j}))$, doing full-wave calculations on the $(\vec{\alpha_j})_{new}$ to get new values of the reflection coefficient functions $g(\omega_i)_{new}$ and $(\partial g_i/\partial\alpha_j)_{new}$, checking the linearity of $g(\omega_i)$ by doing "SETTLE-DOWN" calculations, and finally increasing the value of "λ". The process then continues by following through with the same series of steps again.

The final solution (i.e., final inverted electron density profile) is obtained as α^{final} as shown in figure 8. The definition as to what determines that any given profile in the iterative sequence should be identified as the final profile will be that profile for which the variable S' of equation (63) reaches a turning point (or "elbow") in a plot of S' versus iterative step number.

TABLE I

RATIO THRESHOLDS

RATIO	COMMENTS	$\Delta\lambda$	λ	COMMENTS
0.0 "A" 0.8	Convergence is going too slowly, so want to speed up. Use $\Delta\lambda_{new}$ to get λ_{new}	$\Delta\lambda_{new} = \sqrt{2}\Delta\lambda_{old}$	$\lambda_{new} = \lambda_{old} + \Delta\lambda_{new}$	① Using λ_{new} solve SOLEQS to get $(\Delta\alpha_j)_{new}$. ② Compute new profile $(\alpha_j)_{new}$ ③ Do full-wave on $(\alpha_j)_{new}$
0.8 "B" 1.2	Convergence is going about right, so want to continue at same increment. Use $\Delta\lambda_{new}$ to get λ_{new}	$\Delta\lambda_{new} = \Delta\lambda_{old}$	$\lambda_{new} = \lambda_{old} + \Delta\lambda_{new}$	Same as "A" above.
1.2 "C" 2.0	Convergence is going too fast. Want to slow down. Use $\Delta\lambda_{new}$ to get λ_{new}	$\Delta\lambda_{new} = \Delta\lambda_{old}/2$	$\lambda_{new} = \lambda_{old} + \Delta\lambda_{new}$	Same as "A" above.
2.0 "D"	Results are too far off convergence curve. Will compute	$\Delta\lambda_{new} = \Delta\lambda_{old}/2$ "saved"	$\lambda_{new} = \lambda_{old}$	① Using λ_{old} to solve SOLEQS for $(\Delta\alpha_j)_{new}$ is the same as using $(\Delta\alpha_j)_{new}$

TABLE 1

CONTINUED

RATIO	COMMENTS	$\Delta\lambda$	λ	COMMENTS
4.0	$\Delta\lambda_{new}$, however it will not be used but will be saved for future steps.			from settle down solution of SOLEQS. ② Use $(\Delta\alpha_j)_{settle\ down}$ to get new profile. ③ Do full-wave on $(\alpha_j)_{new}$
4.0 "E" ∞	Same as "D" above	$\Delta\lambda_{new} = \Delta\lambda_{old}/2$ "saved"	$\lambda_{new} = \lambda_{old} - \Delta\lambda_{old}$ $= \lambda_{previous}$	① Set $(\alpha_j)_{new}$ $= (\alpha_j)_{prev.}$ ② Do full-wave on these (α_j)'s

The function S′ is a linear approximation to the function S which is given by:

$$S = \overbrace{g_i - G_i}^{i\rightarrow} \cdot \underset{\downarrow}{i}\left(\frac{1}{\sigma_{ii'}}\right)^{i'\rightarrow} \cdot \underset{\downarrow}{i'}\left(\frac{1}{\sigma_{i'i}}\right)^{i\rightarrow} \cdot \underset{\downarrow}{i}\left(g_i - G_i\right) \tag{66}$$

where the G_i are data values as given by equation (28) and contain some amount of error. Equation (66) is functionally related to equation (6) and replaces the "S" factor in equations (12) and (13).

Figure 9 illustrates the general characteristics of the S′ and S functions as related to the number of the iterative step at any point in the inversion procedure. The simulated data used in figure 9 has error statistics as will be discussed in section V-A. That is the error is gaussian distributed and

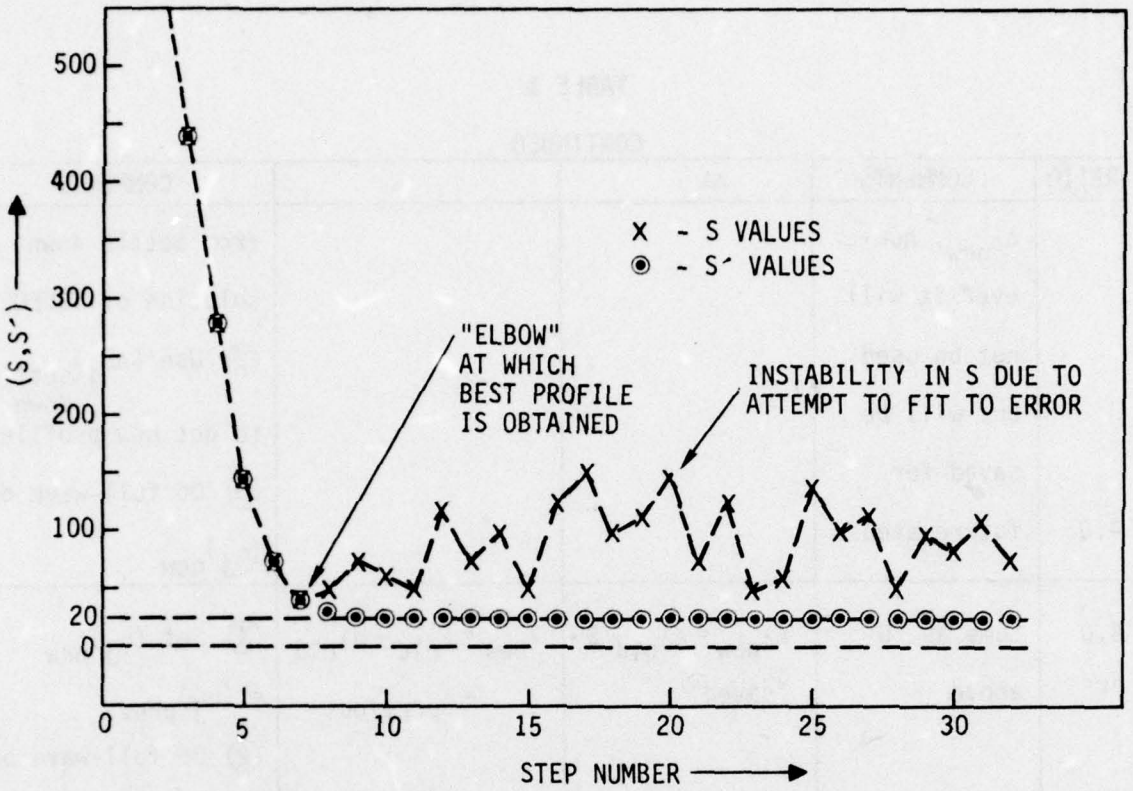


Figure 9. Comparison between S and S' as a Function of Iterative Step Number.

uncorrelated and also the expected error values, σ_i, (i.e., the standard deviation values) are known. For the results presented in figure 9, there are 20 data points used in the computation (i.e., $i = 1, \ldots m$ where $m = 20$). This implies that the lowest value of S (or S') to be expected is approximately 20. The approximate nature of this number is due to the number of data points being finite. That is, for larger sets of data $(S/m) \to 1$ as $m \to \infty$ if the error for each data parameter is independent of the error for any other data parameter. Continuation of the iterative process to obtain S (or S') values much less than 20 represents an unsuccessful attempt to fit the profile to the error in the data. The transition from fitting to data to fitting to the error is illustrated in figure 9 as an "elbow" in the S' curve. Beyond this point oscillations are introduced into the profile in an

attempt to fit to the error which in turn leads to instability in the values of S . The linear projections represented by the values of S′ give an indication of the best fit that can be expected.

Note that the very definite transition point, represented by the "elbow" in figure 9, is the result of complete knowledge of the error statistics. That is, the expected errors (i.e., the standard deviation values, σ_i , which are used with gaussian distributed random numbers to simulate the errors) for this simulated example are known and the error for each data variable, $G(\omega_i)$, is independent of the error for every other data variable. Otherwise, the transition represented by the "elbow" would be much less well-defined and might even be undetectable if too little is known about the actual error statistics.

Experience gained in executing the INVERT program indicates that there is an optimum magnitude of the "SETTLE-DOWN" of the profile which can be tolerated for a valid sequence of profiles. Singularities of the coefficient matrix of equation (41) will lead, however, to ever increasing values of DPRIME as the value of "λ" corresponding to the singularity condition is approached. When this occurs in the inversion sequence, no further progress can be made. This case is illustrated in figure 10. The only possibility for progress is to re-execute the INVERT computer program from the beginning with a new initial value of the input variable, ROTATE, (i.e., the direction of rotation of the $r(\omega_i)$ curve is reversed). If this option is not successful in alleviating the problem, no solution is possible, for the given set of reflection coefficient data, when using the INVERT program.

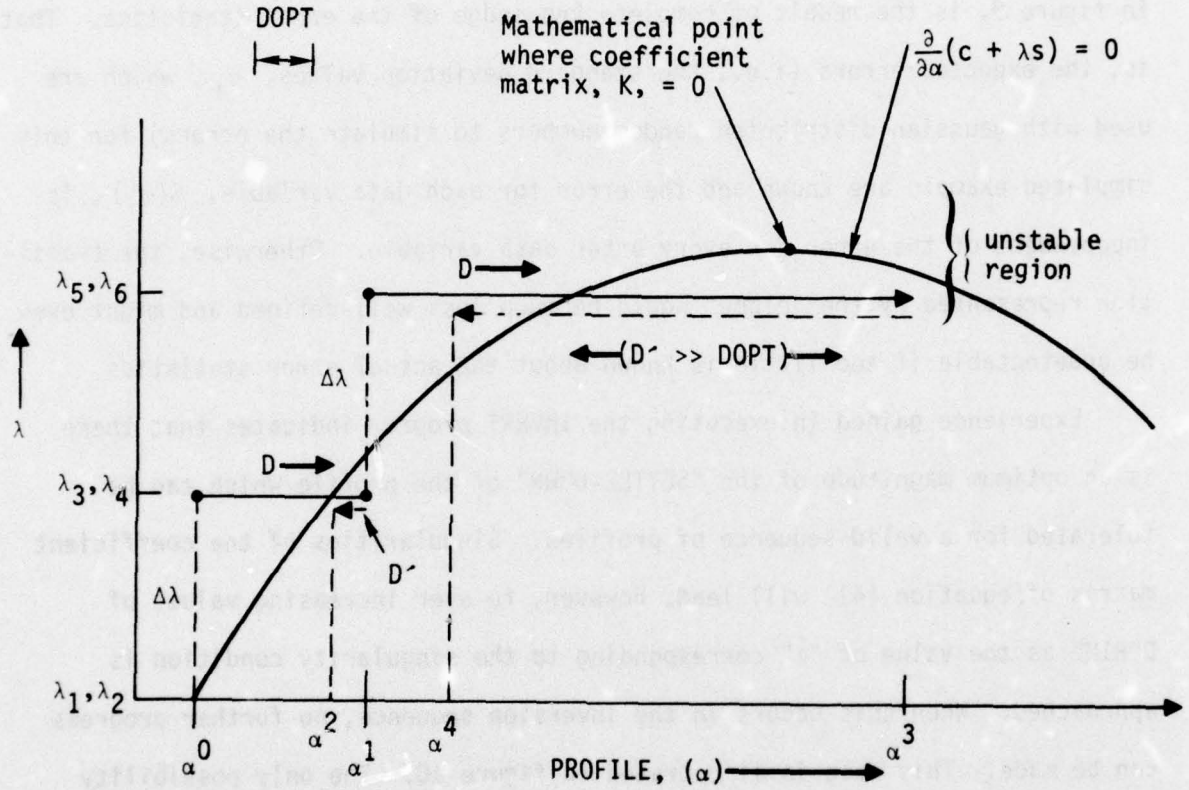


Figure 10. Iterative Sequence in which Divergence Occurs.

IV. STRUCTURE AND DESCRIPTION OF THE INVERT COMPUTER PROGRAM

A. Program Structure

Figure 11 presents the general flow of the INVERT computer program showing the relationship between the various subroutines.

An important limitation built into the INVERT program is that execution is possible for an arbitrary choice of frequencies only as long as the following rule is applied:

$$\left\{ \frac{\text{Frequency Max. - Frequency Min.}}{\text{DELTA Frequency}} \right\} \leq 200 \qquad (67)$$

B. Description of Computer Program Routines

The function of each program subroutine is listed below:

(1) MAIN ROUTINE:

This routine controls the input of inversion parameters through the Namelist, DATUM. It also provides for the input of reflection coefficient data and the input of full-wave parameters.

(2) SUBROUTINE DINPUT:

Input of data: reflection coefficients (R), uncertainties (σ), and propagation frequencies (ω).

(3) SUBROUTINE FWINPT:

Input of full-wave parameters through the Namelist, FULL WV. These parameters include the angle of incidence of the radio wave onto the ionosphere, "THETA", and the geomagnetic parameters of the earth: propagation azimuth, "AZIM", (i.e., degrees east of magnetic north), Codip angle, "CODIP", magnetic field

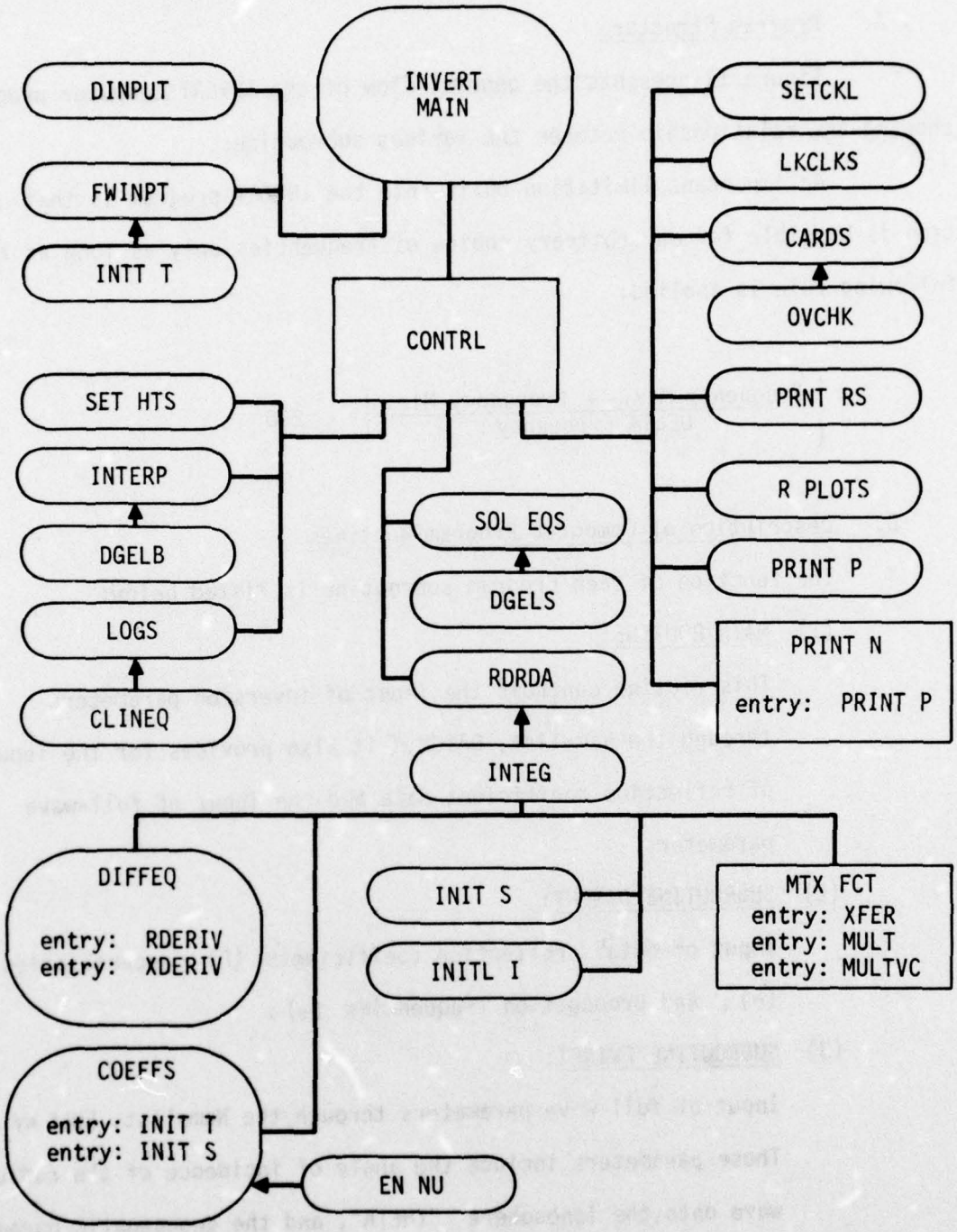


Figure 11. Structure of the INVERT Computer Program.

strength, "MAGFLD", (in Webers/square meter). Also input are the options for obtaining printer plots of electron density profiles, "IPROFP", and reflection coefficient plots, "IRPLOT".

(4) SUBROUTINE CONTRL:

This routine controls the running sequence of the remainder of the total program, especially the incrementing of the trade-off parameter "λ" which determines the convergence of the inversion scheme.

(5) SUBROUTINE SETHTS:

This routine sets up the initial electron density profile. Here, use is made of the input parameters: HTSTOP(1) and HTSTOP(2) (i.e., the profile heights where the maximum and minimum electron densities are assigned. Also of importance is that the value of (HTSTOP(1) - HTSTOP(2))/DELTAH must be a multiple of 2. DELTAH is the height increment between each profile point. See equation (51).

(6) SUBROUTINE INTERP:

This routine sets up coefficients used for interpolation between the given data points of reflection coefficients (R) versus propagation frequency (ω). These coefficients are the elements of the β-matrix of equation (34). This routine uses the subroutine DGELB.

(7) SUBROUTINE DGELB:

The routine solves a system of simultaneous linear equations with a coefficient matrix of band structure, (i.e., $\underset{\sim}{X} = C^{-1} \cdot \underset{\sim}{Y}$ where $\underset{\sim}{C}^{-1}$ is of the form:

$$\underset{\sim}{C}^{-1} = \begin{pmatrix} b & a & & \\ c & b & a & \\ & c & b & a \\ & & c & b \end{pmatrix}$$

(8) SUBROUTINE LOGS:

The routine sets up the parameters for making use of the G function of equation (28). That is for $G_i = \ln (dR/d\omega)_i$ where R_i are the input variables (i.e., functions of the reflection coefficients). The routine generates a continuous curve $R(\omega)$ (e.g., continuous in phase) by making use of the β-matrix of equation (34). Subsets ($a_{ii'}$) of the β-matrix are used to compute the derivative term $(dR/d\omega)_i$ along the $R(\omega)$ curve. This derivative is given by equation (35). Also obtained are the derivatives $(dG/dR)_i$ of equation (47). The uncertainty matrix $(1/\sigma)_{new}$ of equations (44) and (45) are also computed. This routine uses the subroutine CLINEQ.

(9) SUBROUTINE CLINEQ:

This routine finds the solution, $\underset{\sim}{X}$, of the matrix equation $\underset{\sim}{A} * \underset{\sim}{X} = \underset{\sim}{L} * \underset{\sim}{U} * \underset{\sim}{X} = \underset{\sim}{B}$ where $\underset{\sim}{L}$ and $\underset{\sim}{U}$ are triangular matrices.

(10) SUBROUTINE RDRDA:

This routine sets up the parameters for making use of the "g" function of equation (24). It computes the following parameters: $(dr/d\omega)_i$ of equation (37), $(\partial r_i/\partial \alpha_j)$ of

equation (49) and $(\partial g_i/\partial \alpha_j)$ of equation (48) to be used in the subroutine SOLEQS. The continuity of phase for $(dr/d\omega)$ is also accomplished in this routine.

(11) SUBROUTINE INTEG:

This routine does the full-wave integration for the ionospheric reflection coefficients (i.e., $\underset{\sim}{R}$ of equation (1) and $(\partial \underset{\sim}{R}/\partial \alpha_j)$. The integration is accomplished by a Runge-Kutta procedure. INTEG makes use of the following subroutines: INITL I, DIFFEQ and COEFFS.

(12) SUBROUTINE INITL I:

This routine computes the ionospheric reflection coefficient matrix, $\underset{\sim}{R}$, for a sharply bounded ionosphere. These results are used as starting values for the $\underset{\sim}{R}$ integration of subroutine INTEG.

(13) SUBROUTINE COEFFS:

This routine computes the coefficients for $d\underset{\sim}{R}/dz$.

(14) SUBROUTINE INIT T:

This routine computes variables which are used in the full-wave solution and are functions only of the geomagnetic parameters, (AZIM, CODIP and MAGFLD) and the angle of incidence, THETA.

(15) SUBROUTINE INIT S:

This routine computes variables which are used in the full-wave solution and are functions of the variables of INIT T and also of the propagation frequency, ω.

(16) SUBROUTINE DIFFEQ:

This routine computed the derivatives ($d\underset{\sim}{R}/dz$) and $d/dz\ (\partial\underset{\sim}{R}/\partial\alpha(z_j))$ for use in the full-wave integration. $\underset{\sim}{R}$ is the ionospheric reflection coefficient matrix of equation (1).

(17) SUBROUTINE EN NU:

This routine interpolates electron density and collision frequency profiles as a function of ionospheric height.

(18) SUBROUTINE SOL EQS:

This routine obtains the solution of equation (41) in terms of the change in profile ($\Delta\alpha_j$) from given values of derivatives ($\partial g_i/\partial\alpha_j$) and a given value of "$\lambda$". This routine also computes the parameters, "error width" and "resolution". The solution ($\Delta\alpha_j$) is obtained through the use of the subroutine DGELS.

(19) SUBROUTINE DGELS:

Solves the matrix equation (41) for ($\Delta\alpha_j$).

(20) SUBROUTINE CARDS:

"Punched" card output for continuing inversion procedure in a subsequent computer run.

(21) SUBROUTINES SET CKL AND LK CLKS:

Systems routines for keeping track of elapsed time so that output to subroutine CARDS will be obtained before computer run is terminated by exceeding the time limit.

(22) SUBROUTINE R PLOTS:

This routine provides computer printer plots of ionospheric reflection coefficient functions, R (data) and r (computed).

(23) SUBROUTINES PRINT N AND PRINT P:

These routines print and plot the electron density profile of the ionosphere.

(24) SUBROUTINE PRINT RS:

This routine prints summary tables of computed parameters for each successive iteration in the inversion scheme. The printed values are: r_i (real, imaginary), magnitude of $(dr/d\omega)$, phase of $(dr/d\omega)$ in degrees, the transformation function g (real) and g (imag.), in radians.

(25) SUBROUTINE OVCHK:

This routine checks for overflows and underflows during computation.

(26) SUBROUTINE MTX FCT:

This subroutine provides for matrix manipulations.

C. Identification of NAMELIST Variables

(1) NAMELIST: & FULL WV

(a) 'THETA' -- Incident angle of the transmitted wave onto the ionosphere. (degrees)

(b) Geomagnetic parameters of the earth's magnetic field.

'AZIM' -- The clockwise angle between magnetic north and the horizontal propagation direction east of north. (degrees)

'CODIP' -- The magnetic co-dip angle measured from the vertical downward. (degrees) The magnetic equator is specified by 'CODIP' = 90° while the north magnetic pole is identified as 0°.

"MAGFLD" -- The intensity of the earth's magnetic field (Webers/square meters).

(c) "IPROFP" -- Option for obtaining printer plots of a simulated original electron density profile from which reflection coefficients have been previously computed and are to be used as input data to the program "INVERT". If 'IPROFP' = 1, then computer plots of this simulated profile will be superimposed upon plots which are automatically printed of electron densities obtained from successive iterations of the inversion scheme. Note that the profile heights of the simulated profile must be identical to the heights of the profiles obtained through the iterative steps. If not the program execution stops.

If 'IPROFP' = 0, only the iterative profiles are plotted. This is because if the input data values, $R(\omega_i)$, were obtained from actual measurements, there would be no simulated data profile and therefore no profile to plot.

(d) 'RPLOTS' -- Option for obtaining printer plots of r (real) and r (imaginary) in the complex r-plane, 'RPLOTS' = 1 gives the plots, 'RPLOTS' = 0 results in no plots.

(2) NAMELIST: &DATUM

(a) 'HTSTOP' -- The heights (km) of the electron density profile at which the profile "stops" are to be applied. "HTSTOP(1)" is the higher height corresponding to "STOPS(1)" while "HTSTOP(2)" is the lower height corresponding to "STOPS(2)".

(b) 'STOPS' -- The limiting values (in electrons/cm^3) of the electron densities that the iterated profile can take on at the heights. HTSTOP(1) and HTSTOP(2).

(c) 'NUFLAG' -- This parameter allows for a choice of exponential collision frequency profiles to be used in the calculations. The exponential collision frequency is given by $\nu(z)$ = COEFNU * EXPNU(z) where "z" is the ionospheric height in km.

(i) If "NUFLAG = 0", the values are:

COEFNU = 1.816 E + 11, coll/sec

EXPNU = -0.15 km^{-1}

(ii) If "NUFLAG = 1", the values are:

COEFNU = 4.303 E + 11, coll/sec

EXPNU = -0.1622 km^{-1}

(iii) If a different exponential is desired, set "NUFLAG = 0" and enter the new values of COEFNU and EXPNU through the NAMELIST, DATUM.

(d) 'DELTAL' -- The initial value of the parameter $\Delta\lambda$.

(e) 'DOPT' -- A term used in the iterative procedure and defined by:

$$\text{RATIO} = \frac{\text{DPRIME}}{\text{DOPT}}$$

(f) 'ALPHA' -- The values of the logs of electron densities at each height of the profile, (i.e., $\alpha(z)_j = \log_{10} N_e(z)$). These ALPHA values are obtained from a previous execution of the INVERT program and are output from the subroutine "CARDS".

(g) 'NRSTPS(J,I)' -- The number of Runge-Kutta steps in a pair of height intervals between heights $(h_{(2j-1)})$ and $(h_{(2j+1)})$ for the i-th frequency, ω_i.

(h) 'NR' -- The number of increments that have been made in "λ" in solving equation (41).

(i) 'IPRNTE' -- The truncation error in Runge-Kutta analysis is printed when IPRNTE = 1.

(j) 'IPRINT' -- Printout of Budden's reflection coefficients as a function of height. Printed when IPRINT = 1.

(k) 'ROTATE' -- Determines the rotation of the computed $r(\omega)$ curve in the r-plane. The values used are that:

ROTATE
- = "+", is counter-clockwise rotation.
- = "-", is clockwise rotation.
- = "0", the program chooses the rotation direction of smallest difference between the ϕ_D and the ϕ_C of section (II-I).

The following tables give the initial values of the NAMELIST variables.

TABLE II

NAMELIST VARIABLES AND INITIAL VALUES,

NAMELIST: &FULL WV

NAME	VALUE	UNITS
AZIM	90.0	degrees
CODIP	12.0	degrees
MAGFLD	5.3 E - 5	Weber/square-meter
THETA	22	degrees
IRPLOT	1	none
IPROFP	1	none
IPRINT	0	none

TABLE III

NAMELIST VARIABLES AND INITIAL VALUES,

NAMELIST: &DATUM

NAME	VALUE	UNITS	NAME	VALUE	UNITS
ALAMDA	0.0	km	HTSTOP(1)	93	km
ROTATE	0.0	degrees	HTSTOP(2)	50	km
DELTAH	1.0	km	STOPS(1)	1.0 E 3	electrons/cm^3
DELTAL	1.0/64.0	km	STOPS(2)	3.0	electrons/cm^3
DELF	0.1	kHz	COEFNU	1.816 E 11	collisions/sec
DOPT	1.0	none	EXPNU	-0.15	km^{-1}
IPRNTE	0	none	ALPHA(51)	none	none
NUFLAG	0	none	NRSTPS	none	none
NR	1.0	none			
MIN	10.0	minutes			

V. EXECUTION OF INVERT USING SIMULATED DATA

A. Errors

To illustrate the inversion procedure, the INVERT computer program may be executed using simulated reflection coefficient data as input.

The simulated data input is described by:

$$R(\omega_i)_\varepsilon = R(\omega_i)_{F.W.} + \varepsilon_i \qquad (68)$$

$$i=1,\ldots m$$

where

ω_i -- are the propagation frequencies.

$R(\omega_i)_{F.W.}$ -- are the reflection coefficient variables as defined by equation (2) and are obtained from execution of the FULLWAVE computer program. These values may be thought of as the true values of the Reflection coefficients of the data when no error is present.

ε_i -- are the actual simulated errors assumed to be present in the data $R(\omega_i)_\varepsilon$ at each frequency.

The above variables are computed using two auxiliary computer programs. These are identified as FULLWAVE and RANDOM. FORTRAN listings of these programs are included in appendices A and B of this report.

B. The Auxiliary Program, FULLWAVE

The FULLWAVE computer program gives simulated reflection coefficient values for the factor $R(\omega_i)_{F.W.}$ of equation (68). These values are obtained from the integration of equation (4) and the generation of equation (5) (or equation (2)). The input to the program is a simulated electron density profile (i.e. the "Data Profile"), assumed collision frequency profile, geomagnetic field parameters, the frequency range desired and a value to be assigned to the expected error, SIGMA.

Figure 12 illustrates the relationship of the various subroutines for the FULLWAVE program.

The following is a list of the subroutines used in the FULLWAVE computer program.

(1) MAIN ROUTINE:

Provides for input to the program, including input electron density and collision frequency profile. It also provides for NAMELIST input.

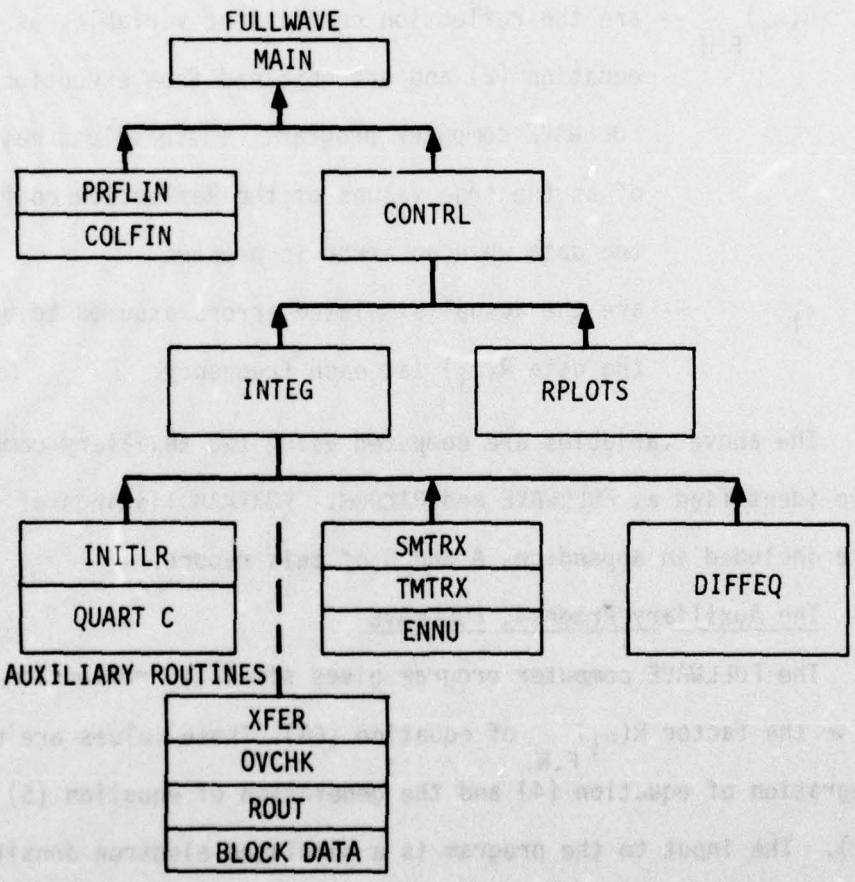


Figure 12. Program Flow for FULLWAVE.

(2) SUBROUTINE PRFLIN:

Provides for reading an electron density profile into the program. This profile is a table in terms of height and electron/cm^3.

(3) SUBROUTINE COLFIN:

Provides for reading a collision frequency profile into the program. This profile consists of only the frequencies at the top and bottom heights of the electron density profile.

(4) SUBROUTINE CONTRL:

Controls the running sequence of the total program.

(5) SUBROUTINE INTEG:

Performs the integration of the differential equations for the ionospheric reflection matrix using Runge-Kutta integration formulas. Subroutines utilized for this operation are: INITLR, QUARTC, SMTRX, TMTRX, ENNU, DIFFEQ.

(6) SUBROUTINE RPLOTS:

Provides for printer plots of real and imaginary parts of the reflection coefficient function of equations (2) and (5).

(7) AUXILIARY ROUTINES:

BLOCK DATA, XFER, OVCHK and ROUT.

The following are NAMELIST, "DATUM" input variables:

(1) FLOW:

The lowest frequency (kHz) for which full-wave computations are to be made.

(2) FHI:

The highest frequency (kHz) for which full-wave computations are to be made.

(3) FINC:

The increment in frequency (kHz).

(4) THETA:

The angle of incidence on the ionosphere of the transmitted radio wave.

(5) AZIM:

The azimuth of the radio wave direction measured in degrees east of magnetic north.

(6) CODIP:

The codip of the magnetic field vector. CODIP = 90° at the earth's magnetic equator and = 0° at the earth's north magnetic pole.

(7) MAGFLD:

The magnitude of the earth's magnetic field (webers/m^2)

(8) NUFLAG:

A flag for determining the collision frequency profile to be used in the "full wave" computations. The following criteria is used:

NUFLAG = 0, COEFNU = 1.816E11 collisions/sec.
EXPNU = -0.15 km^{-1}

NUFLAG = 1, COEFNU = 4.303E11 collisions/sec
EXPNU = -0.1622 km^{-1}

If a table of collision frequency values is to be read in through the subroutine COLFIN, then NUFLAG must also be set to zero.

(9) NPUNCH:

If NPUNCH = 0, no output cards are punched. If NPUNCH = 1, output cards are obtained with frequency (kHz), real and imaginary parts of the R variable of equation (2) and a choice for the expected error, SIGMA, on each card.

(10) IPROFP:

Provides for printer plot of electron density profile:

IPROFP = 1: plot is obtained

IPROFP = 0: no plot

(11) IRPLOT:

Provides for printer plot of R-parameter from equation (2).

IRPLOT = 1: plot is obtained

IRPLOT = 0: no plot

(12) SIGMA:

The value of the expected error, σ_{ii}, assigned to each data value, $R(\omega_i)$.

TABLE IV

INITIAL VALUES OF FULLWAVE

NAMELIST VARIABLES

NAME	INITIAL VALUE	UNITS
FLOW	0.5	kHz
FHI	25.0	kHz
FINC	0.5	kHz
THETA	0.0	degrees
AZIM	none	degrees
CODIP	none	degrees
MAGFLD	none	webers/meter^2
NUFLAG	0	none
COEFNU	1.816×10^{11}	collisions/sec.
EXPNU	-0.15	km^{-1}
NPUNCH	1	none
IPROFP	1	none
IRPLOT	1	none
SIGMA	20(0.03)	none

C. The Auxiliary Program, RANDOM

The computer program RANDOM provides for gaussian random error values, ε_i, to be added to the FULLWAVE reflection coefficient values $R(\omega_i)_{F.W.}$.

The actual simulated error, ε_i, is computed by the expression:

$$\varepsilon_i = u_{iq} \times \sigma_i \qquad (69)$$

$$i=1,\ldots m$$

where

σ_i -- is the expected error in the data as chosen for each frequency, ω_i.

u_{iq} -- is obtained from a random number generator where "q" is the number identifying the particular set of random numbers used.

Define a random variable u(x), which is gaussian distributed with standard deviation, $\sigma = 1$, as:

$$u(x) \approx \frac{1}{\sqrt{2\pi}} \exp(-x^2/2) \qquad (70)$$

This distribution is shown in figure 13 where the distribution is truncated between -6 and +6.

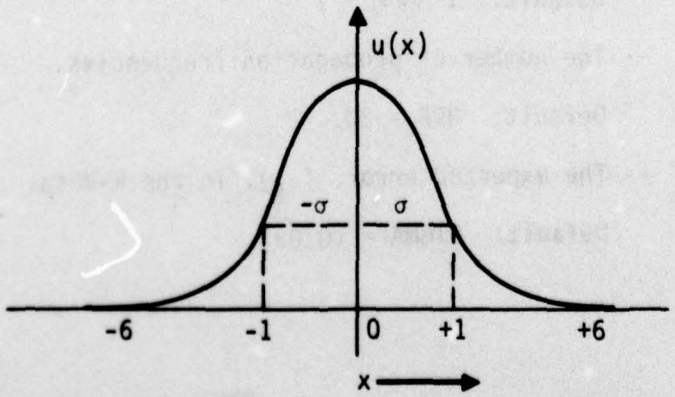


Figure 13. Gaussian Distribution.

Let different sets of random numbers be identified as q = 1,2---, where the value assigned to "q" is input to the program, RANDOM by the NAMELIST variable, IRANDM. A given q-th set of random numbers is given by the values, u_{iq}, i=1,... .

Note that the value chosen for σ_i represents the standard deviation of $\varepsilon_i = u_{iq} \times \sigma_i$ as a large number of sets, q, are considered.

In the computer program, RANDOM, the complex values of $R(\omega_i)_{F.W.}$ are divided into real and imaginary parts. The same value, σ_i, is then applied to each of the parts (i.e. real and imaginary) of a given $R(\omega_i)_{F.W.}$. Separate values of u_{iq} are applied, however, to the real part of $R(\omega_i)_{F.W.}$ and to the imaginary part of $R(\omega_i)_{F.W.}$. In the complex R plane the expected error, σ_i, is represented as a circle of radius σ_i about the data point $R(\omega_i)_\varepsilon$.

The above definitions are such that for a given frequency, ω_i, and given chosen value for the expected error, σ_i, then the true value, $R(\omega_i)_{F.W.}$, will lie within the error circle of radius σ_i, drawn around the value $R(\omega_i)_\varepsilon$, 68.3% of the time as a large number of error sets, q, are considered. This result is illustrated in figure 14.

The NAMELIST variables of the computer program RANDOM consist of the following:

IRANDM -- Determines a specific set of random numbers.

Default: IRANDM = 1.

NRF -- The number of propagation frequencies.

Default: NRF = 20.

SIGMA -- The expected error, (σ_i), in the R-data.

Default: SIGMA = (0.03).

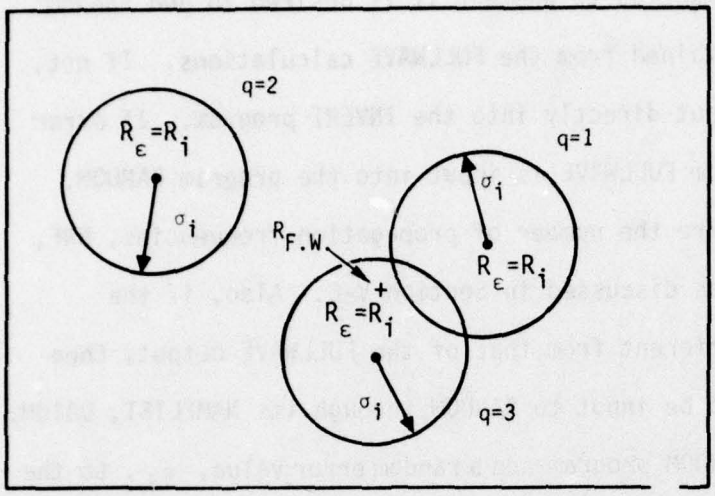


Figure 14. "Error Circles" for a given Frequency, ω_i and given expected error, σ_i, for three error sets, q.

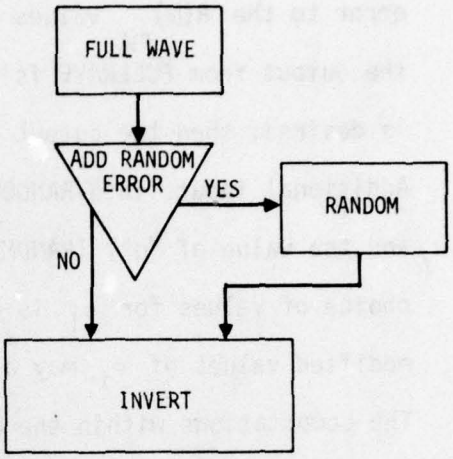


Figure 15. Program flow for using simulated data in INVERT program.

D. Examples of INVERT Execution for Simulated Data

The following sequence of computer runs must be carried through in order to complete an execution of the INVERT program using simulated data. Figure 15 shows the sequence of the required programs.

First, the set of reflection coefficient values, $R(\omega_i)_{F.W.}$ is obtained from the auxiliary program, FULLWAVE. The inputs to this program are the known electron density profile (i.e. simulated data profile), incidence angle, THETA, the geomagnetic field parameters and the range of propagation frequencies desired. The chosen expected error, SIGMA (σ_i), may also be input. There are, however, default values of σ_i incorporated into the program. The output consists of the set of values: frequency, ω_i, computed values of $R(\omega_i)_{F.W.}$ {real} and $R(\omega_i)_{F.W.}$ {imaginary} along with values of the expected error, SIGMA (i.e.σ_i). There is one set of all the above values for each frequency, ω_i.

Second, a choice is made as to whether it is desired to add random error to the $R(\omega)_{FW}$ values obtained from the FULLWAVE calculations. If not, the output from FULLWAVE is input directly into the INVERT program. If error is desired, then the output from FULLWAVE is input into the program RANDOM. Additional inputs into RANDOM are the number of propagation frequencies, NRF, and the value of "q", IRANDM, as discussed in Section V-C. Also, if the choice of values for σ_i is different from that of the FULLWAVE output, then modified values of σ_i may also be input to RANDOM through its NAMELIST, DATUM. The computations within the RANDOM program add a random error value, ε_i, to the FULLWAVE values, $R(\omega_i)_{FW}$, to give the data values, $R(\omega_i)_{\varepsilon}$. The output from the RANDOM program consists of the set of values: frequency, ω_i, computed values of $R(\omega_i)_{\varepsilon}$ {real} and $R(\omega_i)_{\varepsilon}$ {imaginary} along with values of the expected error, SIGMA (i.e., σ_i). There is one set of the above values for each frequency, ω_i.

Finally, the input to the INVERT program consists of the known electron density profile (i.e., simulated data profile), the output values from either FULLWAVE or RANDOM, the incidence angle (THETA), the geomagnetic field variables, and various constraints which are applied in the inversion procedure. The output from INVERT is the inverted electron density profile.

The following examples of input-output values illustrate the inversion sequence for simulated data.

Example I lists the input values to the FULLWAVE program including the profile of electron density per cm^3 at each height and the NAMELIST variables. Example II shows the printed output from FULLWAVE consisting of a table and corresponding plot of the input profile. Example III gives the printed output from FULLWAVE. This includes a table of the values of the output variables and a plot of the $R(\omega_i)_{FW}$ data values in the complex plane. Note that the table

values are also punched on cards to be used as input into either the RANDOM program or directly into INVERT if the no-error case is desired.

Example IV illustrates the input values into the RANDOM program. Note that IRANDM = 2 for this case. Example V gives the printed input. Example VI presents the random numbers computed using IRANDM = 2. These numbers are used to compute the actual simulated error, ε_i. The equation is:

$$\left\{\begin{array}{l} R(\omega_i)_{\varepsilon}\{real\} = R(\omega_i)_{FW}(real) + (U_{i,2}(real) \times \sigma_i) \\ \\ R(\omega_i)_{\varepsilon}\{imag\} = R(\omega_i)_{FW}\{imag\} + (U_{i,2}(imag) \times \sigma_i) \end{array}\right\} \qquad (71)$$

Using the values for ω_1 of example V to illustrate the computation gives:

$$\left\{\begin{array}{l} R(\omega_1)_{\varepsilon}\{real\} = 0.290 + (-3.233 \times 0.03) \\ \qquad\qquad\qquad = 0.193 \\ \\ R(\omega_1)_{\varepsilon}\{imag\} = -0.332 + (1.687 \times 0.03) \\ \qquad\qquad\qquad = -0.281 \end{array}\right\} \qquad (72)$$

The results of these computations are given in example VII. The printed and punched output showing final results of the RANDOM calculations are presented in example VIII. This punched output is to be used as input to INVERT.

In order to show a comparison between the results from two different choices of sets of random errors to be added to the FULLWAVE output, a computation for IRANDM = 3 is also presented in examples IX, X and XI.

The series of input-output examples characteristic of INVERT execution are presented as follows. Example XII gives the input values to INVERT. These values are for the reflection coefficient data containing random error set number 2. The input values are for the variables: propagation frequency ω_i, the data values $R(\omega_i)_\varepsilon$ {real}, $R(\omega_i)_\varepsilon$ {imag} and the expected error, σ_i. Also included are the two sets of NAMELIST variables. These are identified as FULLWV and DATUM. Example XIII shows the printout of input data to INVERT.

Example XIV is the printout of initial steps in the INVERT execution. The following examples are to be compared closely with the discussion concerning figure 8 in section III of this report. Example XV is obtained from the INVERT subroutine PRINTN. Example XVI shows the computed results of $r(\omega_i)$, $|(dr/d\omega)_i|$, $\phi(dr/d\omega)_i$, $|g|$ and ϕ_g as given by equations (5), (25) and (37) as obtained from the full-wave integration of the profile in example XV. Example XVII is the plot of $R(\omega)_\varepsilon$ and $r(\omega)$ values as presented in the complex R (or r) plane. The results of $R(\omega_i)_\varepsilon$ are those listed under "data values" for "RE(R)" and "IM(R)" in example XIV. The inversion values, $r(\omega_i)$, as obtained from full-wave integration of the initial profile of example XV are listed as RE(R) and IM(R) in example XVI and identified as "computed values" in example XVII. Note the large mismatch between the $R(\omega_i)_\varepsilon$ data values and the $r(\omega_i)$ computed values shown in example XVII.

As discussed in section III, the inversion procedure contains a checking criteria which aids in preventing divergence by limiting the size of the increments allowed for the variable "$\Delta\lambda$". This criteria is identified as the "SETTLE-DOWN". Example XVIII shows the "SETTLE-DOWN" printout including the variables: S′ from equation (63), DPRIME from equation (64), RATIO from equation (65) along with a table and printer plot of the "SETTLE-DOWN" profile as obtained from the subroutine PRINTP.

Note in the actual computer printout two additional variables are also present. These are printed as "UNC" (i.e., the uncertainty) and "RES" (i.e., the resolution). These variables are presented and discussed in reference 5 pp. 25-28.

According to the iterative sequence described in section III, the inversion procedure continues as shown in example XIX for steps (a) through (m). That is a new $\Delta\lambda$ (i.e., $\Delta\lambda_3$) is computed giving a new value for "λ" (i.e., λ_3). Next a solution for a new profile increment $(\Delta\alpha)_j$, (i.e., $(\Delta\alpha_j)_5$) is obtained by solving equation (41) with the SOLEQS routine. The profile increment is added to the previous profile giving the new profile α_j^3. Fullwave solutions of $r(\omega_i)\{\text{real}\}$ and $r(\omega_i)\{\text{imag}\}$ are obtained for the profile α_j^3. These $r(\omega_i)$ values are then plotted in the complex r-plane, as in example XVII. The value of "S", from equation (66), is also computed at this point. The SETTLE-DOWN criteria is then applied with results computed for S', from equation (63), DPRIME, from equation (64) and RATIO, from equation (65). The iterative sequence then proceeds again in a similar manner through the steps (a) through (m) with the calculation of a new $\Delta\lambda$ (i.e., $\Delta\lambda_5$) and new "λ" (i.e., λ_5). The complete "boot-strap" procedure is repeated several times with computer plots of each iterative profile and of each match between $R(\omega_i)_\epsilon$ data and the resulting fullwave values of $r(\omega_i)$ presented at each step.

If the resulting values of S', from equation (63), and of S, from equation (66) are manually plotted against iterative step number, the results presented in figure 16 are obtained. As discussed previously in section III, with regard to figure 9, an "elbow" is obtained for the S' curve. Beyond this step oscillations are introduced into the profile by an attempt to fit to the error in the $R(\omega_i)_\epsilon$ values. This in turn leads to instability in the values of S and this instability is shown in the figure.

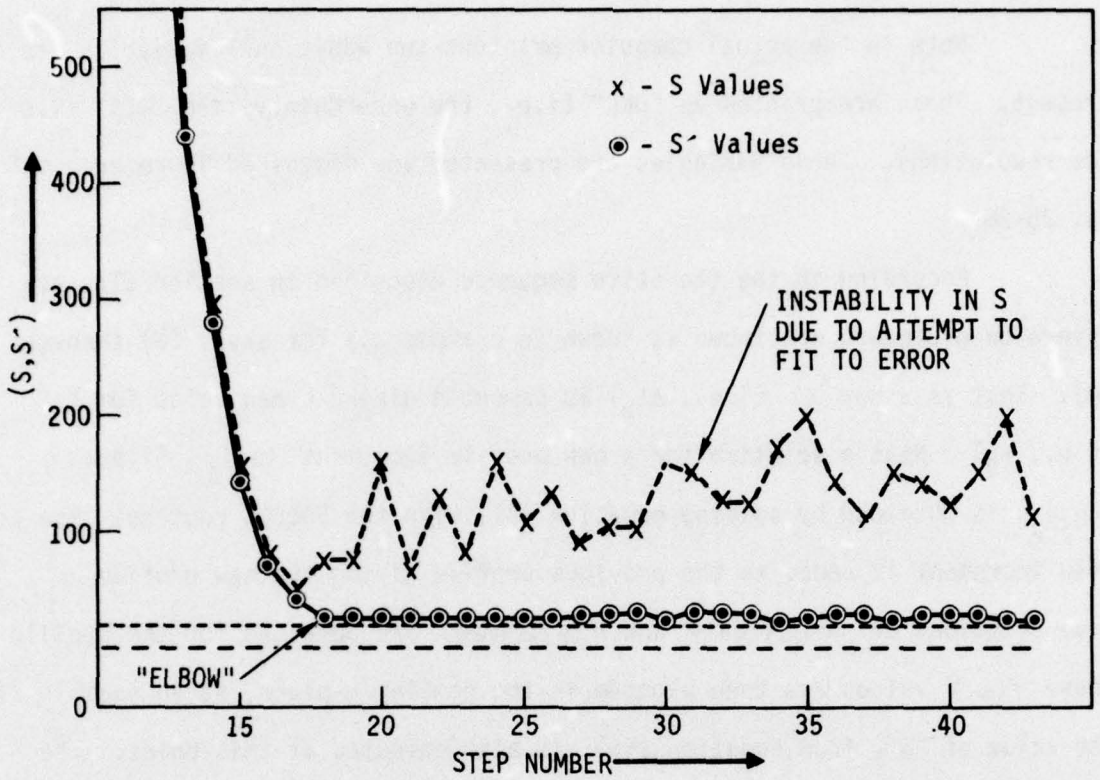


Figure 16. S and S´ Curves for Random Error Set Number 2.

Figure 17 illustrates the inversion results in terms of the S and S´ curves for the set of random errors obtained from examples IX, X and XI, (i.e., IRANDM = 3). Note that these results are qualitatively similar to those of figure 16 but quantitatively different because of the different sets of random errors present. As a matter of interest, figure 18 is presented to show the relationship between the S and S´ curves when no random error is present.

The manual plotting of the S and S´ results, as given by figure 16, allows for the selection of a particular iterative step (i.e., that step number at the "elbow" position) to be chosen as the one which gives the required inverted profile of electron density. For the case presented above, for the random error set number 2, the profile for step number 18 is shown in example XX. The plot compares the original simulated data profile with the chosen final

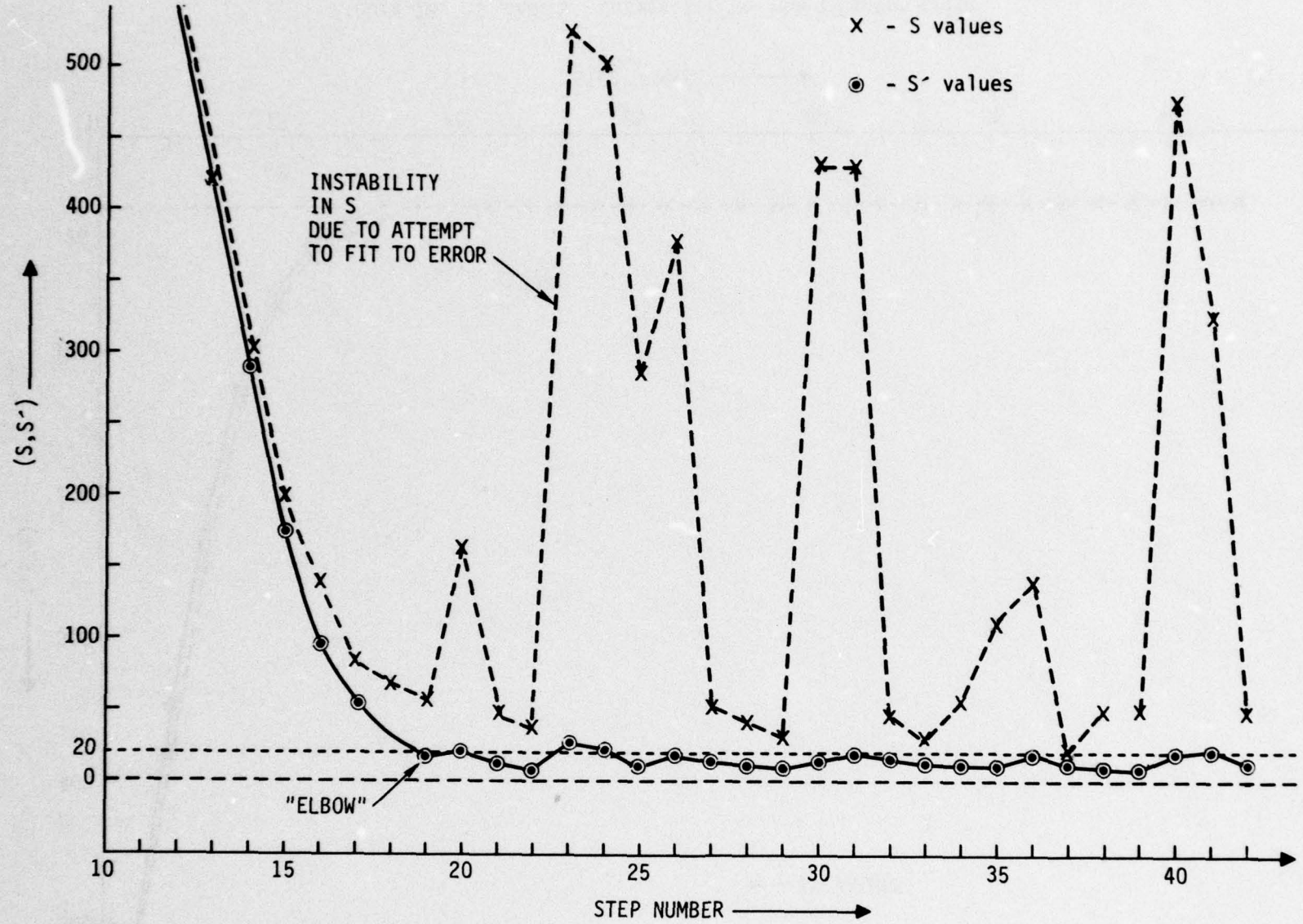


Figure 17. S and S´ Curves for Random Error Set Number 3.

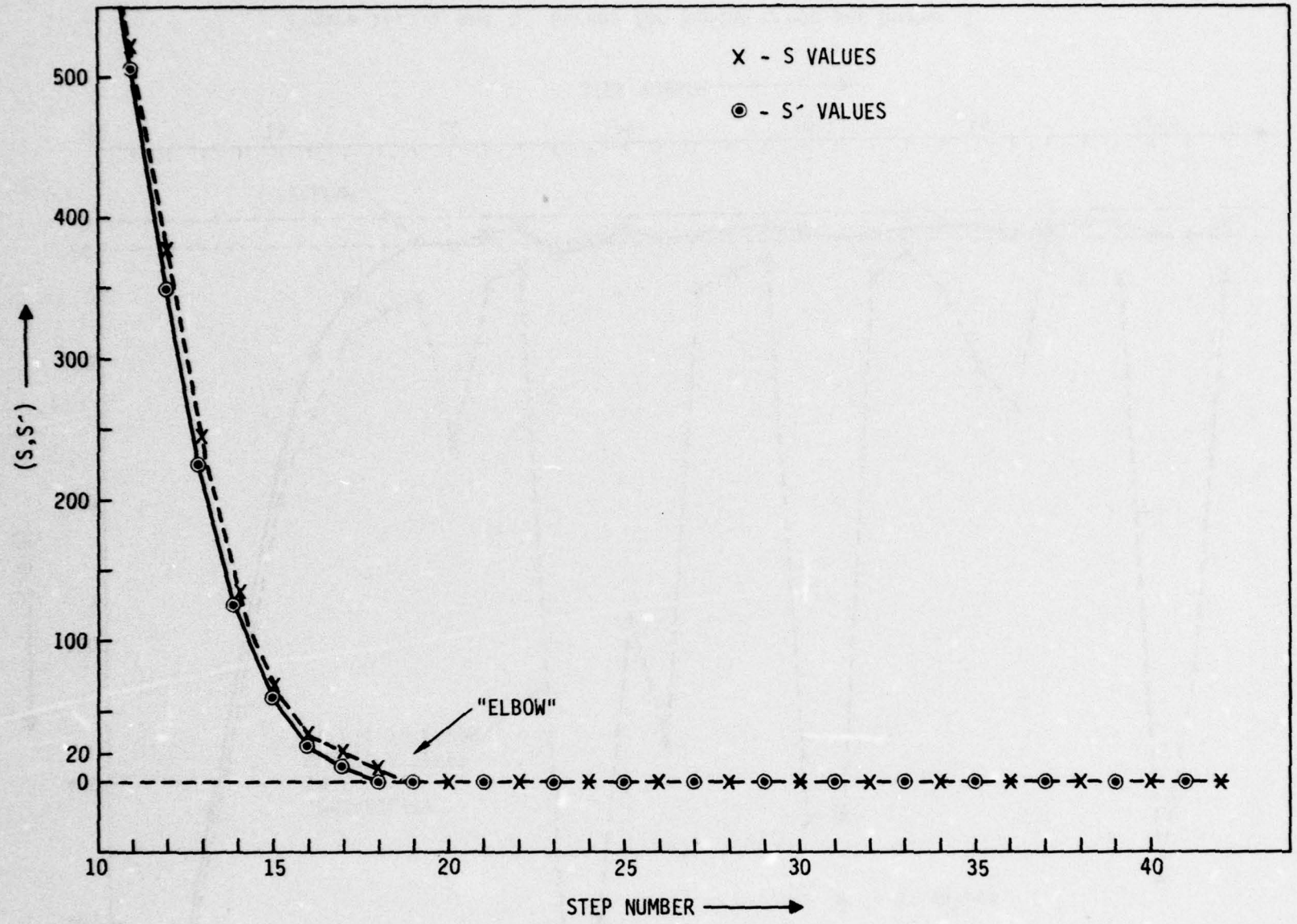


Figure 18. S and S′ Curves for No Added Random Error.

inverted profile. Example XXI illustrates the R-plane comparison between the $R(\omega_i)_\varepsilon$ data points and the full-wave computed values $r(\omega_i)$ using the above inverted profile. Also shown in this example are the printouts obtained from the INVERT calculations.

Section V-C states that for a given frequency, ω_i, and given chosen value for the expected error, σ_i, the true value of $R(\omega_i)_{FW}$ will lie within the "error circle" of radius σ_i, drawn around the value $R(\omega_i)_\varepsilon$, 68.3% of the time for a large number of error sets. The results of example XXI are shown plotted, with these "error circles", in figure 19.

For the random error set number 3 (see examples IX, X and XI), the S and S′ results were shown plotted in figure 17. The inverted profile for step number 19 is shown in example XXII. This plot compares the original simulated data profile with the chosen final inverted profile. Example XXIII illustrates the R-plane comparison between the $R(\omega_i)_\varepsilon$ data points and the full-wave computed values, $r(\omega_i)$, using the inverted profile of example XXII as input. The results of example XXIII are also shown plotted with "error circles" in figure 20.

In figure 18, for the no error case, the scale used for the S and S′ axis was such that the magnitudes of S and S′ appear to be zero at step number 19 and beyond. Figure 21 shows the same plot as figure 18 except on a much expanded scale. In this case the existence of truncation error in the Runge-Kutta integration procedures is obvious. Examination of the results, shown plotted in figure 21, give the choice of integrated profile to be that represented by step number 28. This profile is presented in example XXIV as compared with the original simulated data profile. Example XXV then gives the R-plane comparison between the data points, $R(\omega_i)_{FW}$ and the full-wave computed values, $r(\omega_i)$ from the inverted profile of step number 28.

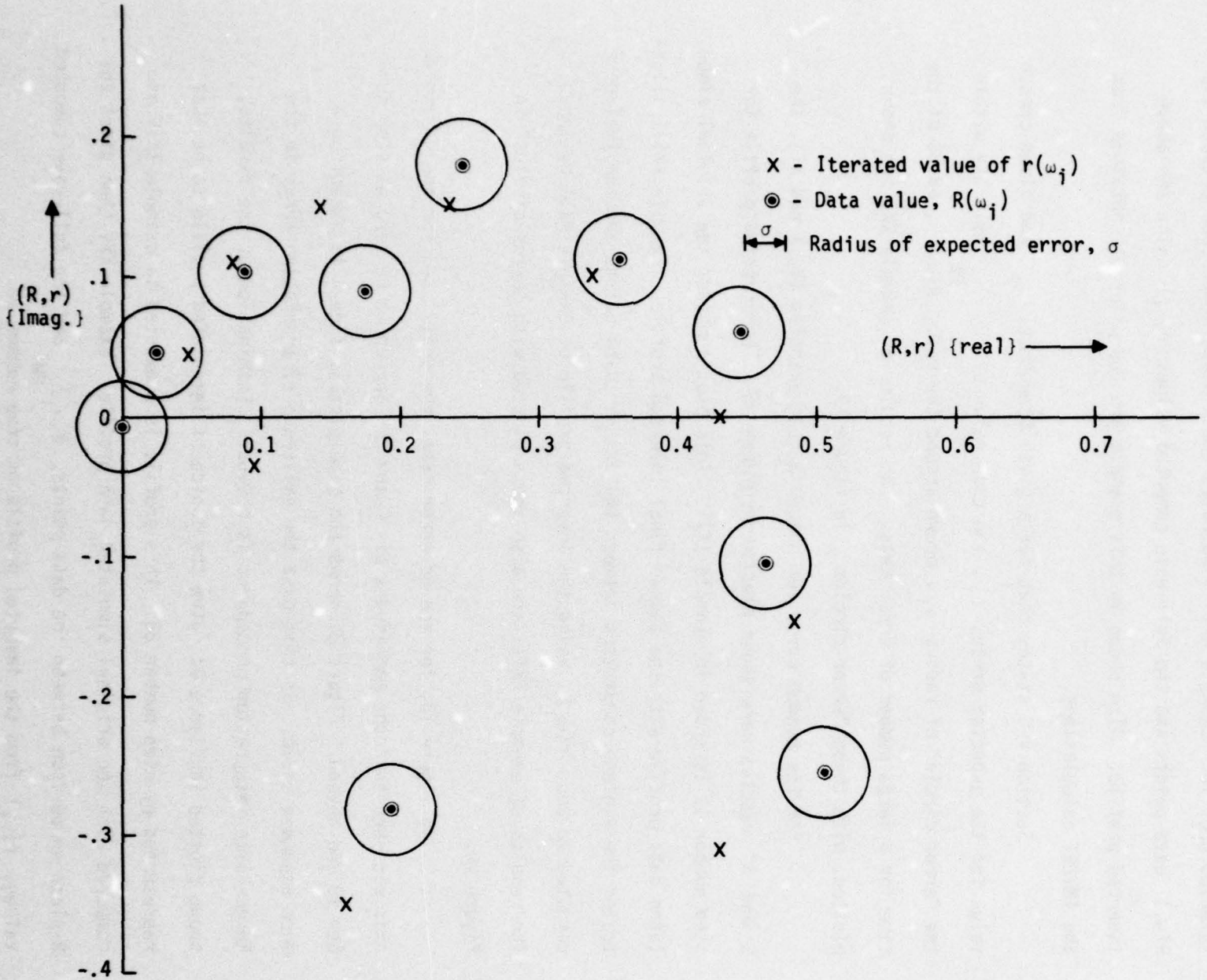


Figure 19. R, r-plane Plot for Random Error Set No. 2 — with Error Circles.

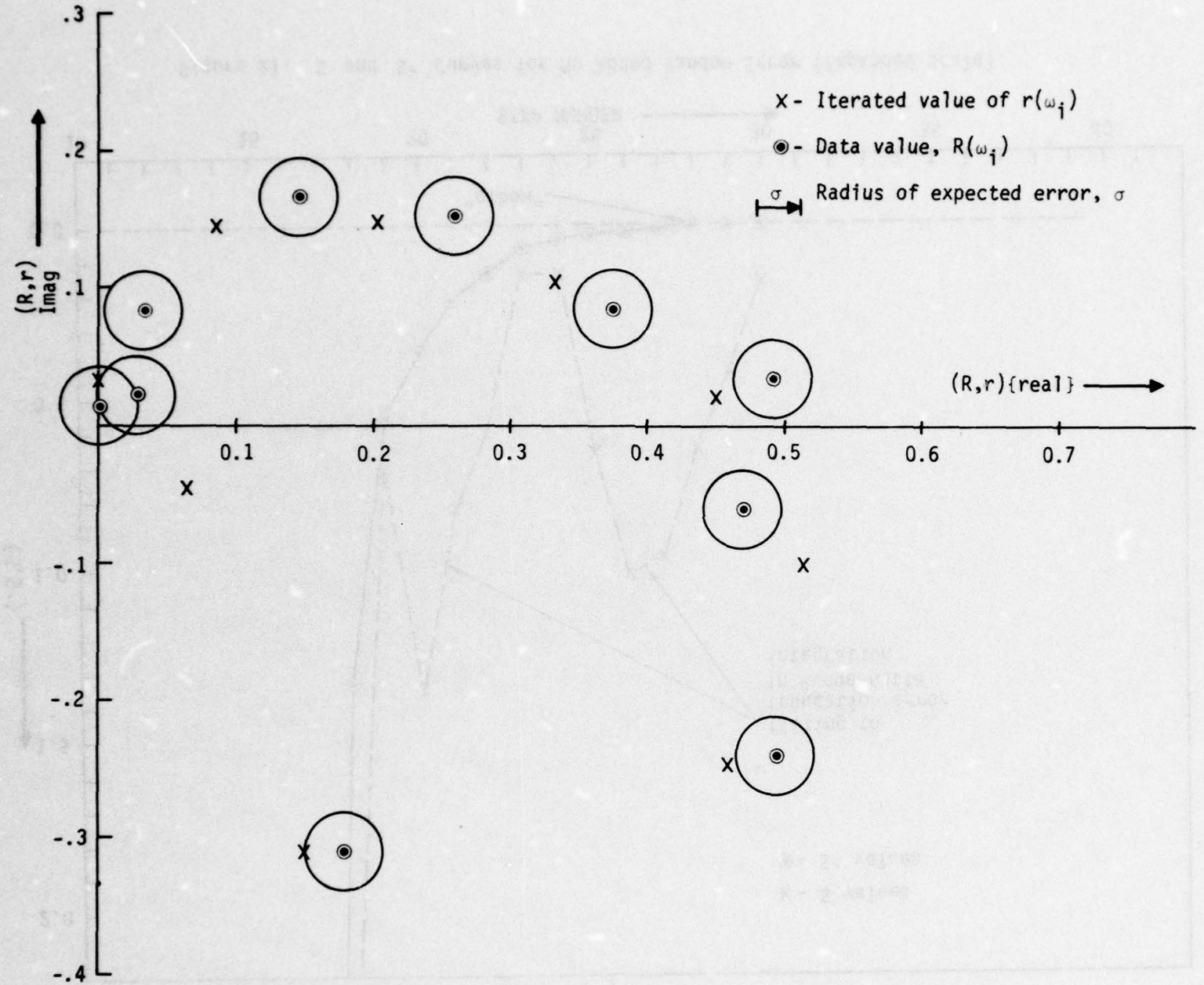


Figure 20. R, r-plane Plot for Random Error Set No. 3 — with Error Circles.

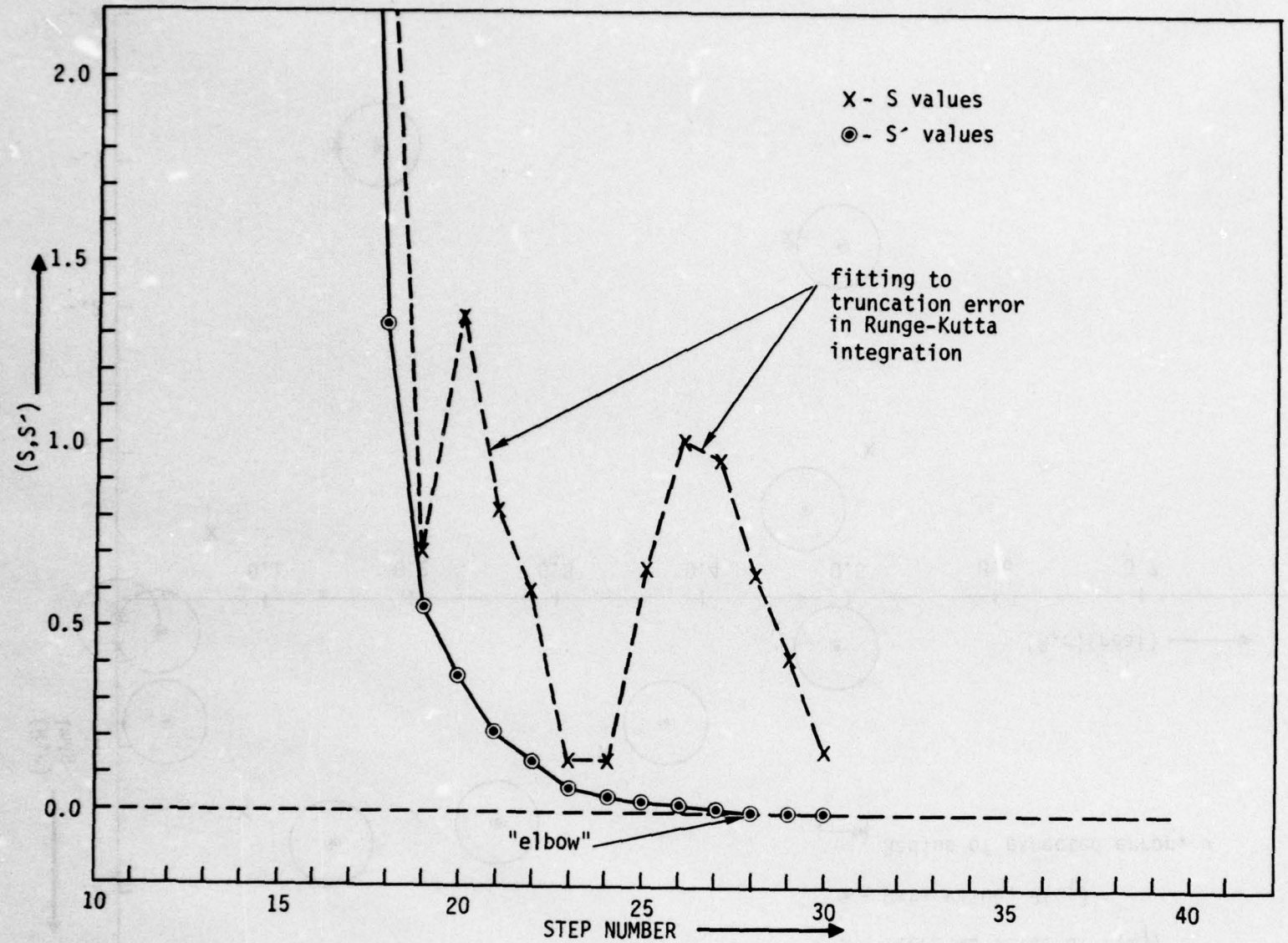


Figure 21. S and S' Curves for No Added Random Error (Expanded Scale).

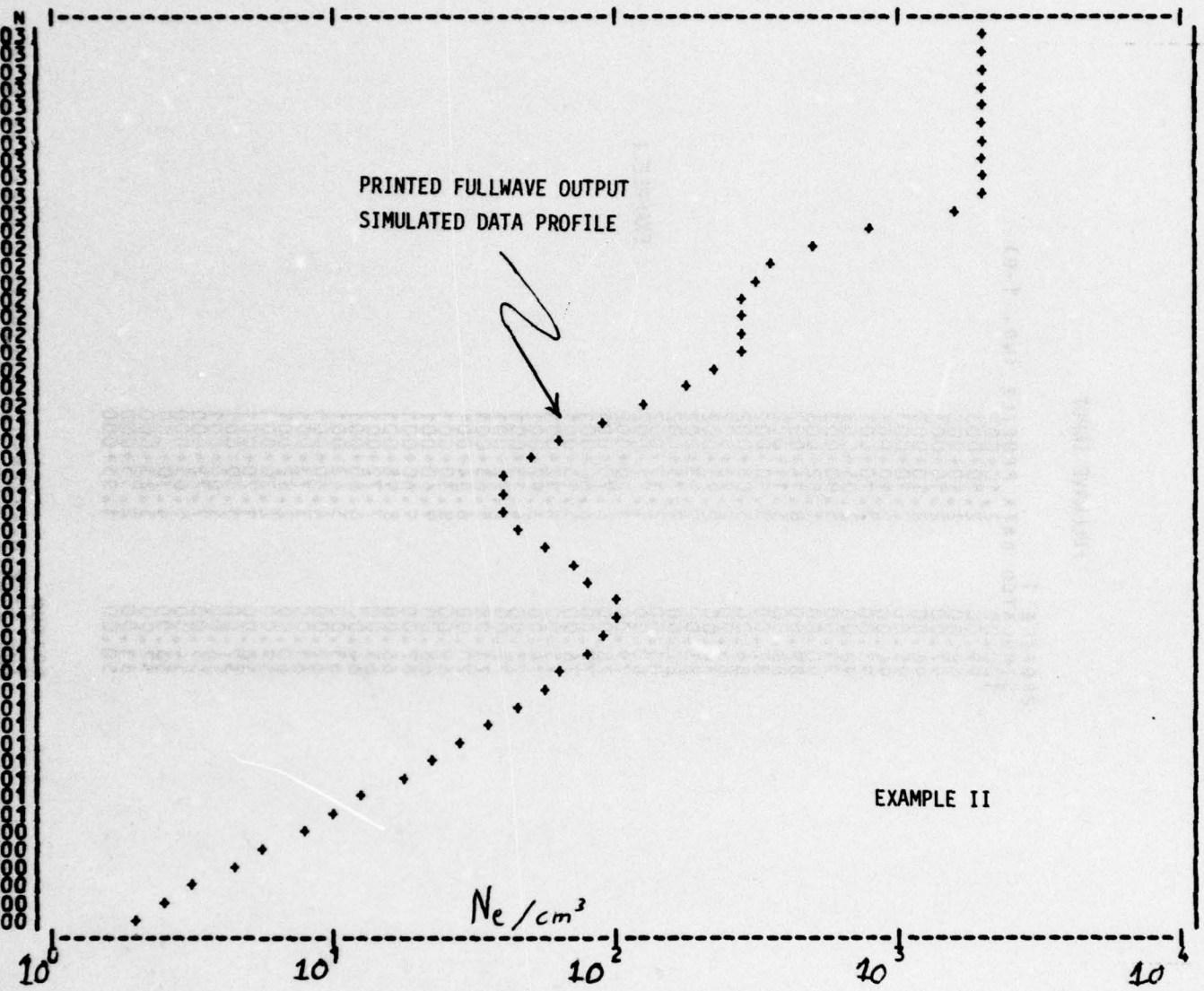
FULLWAVE INPUT

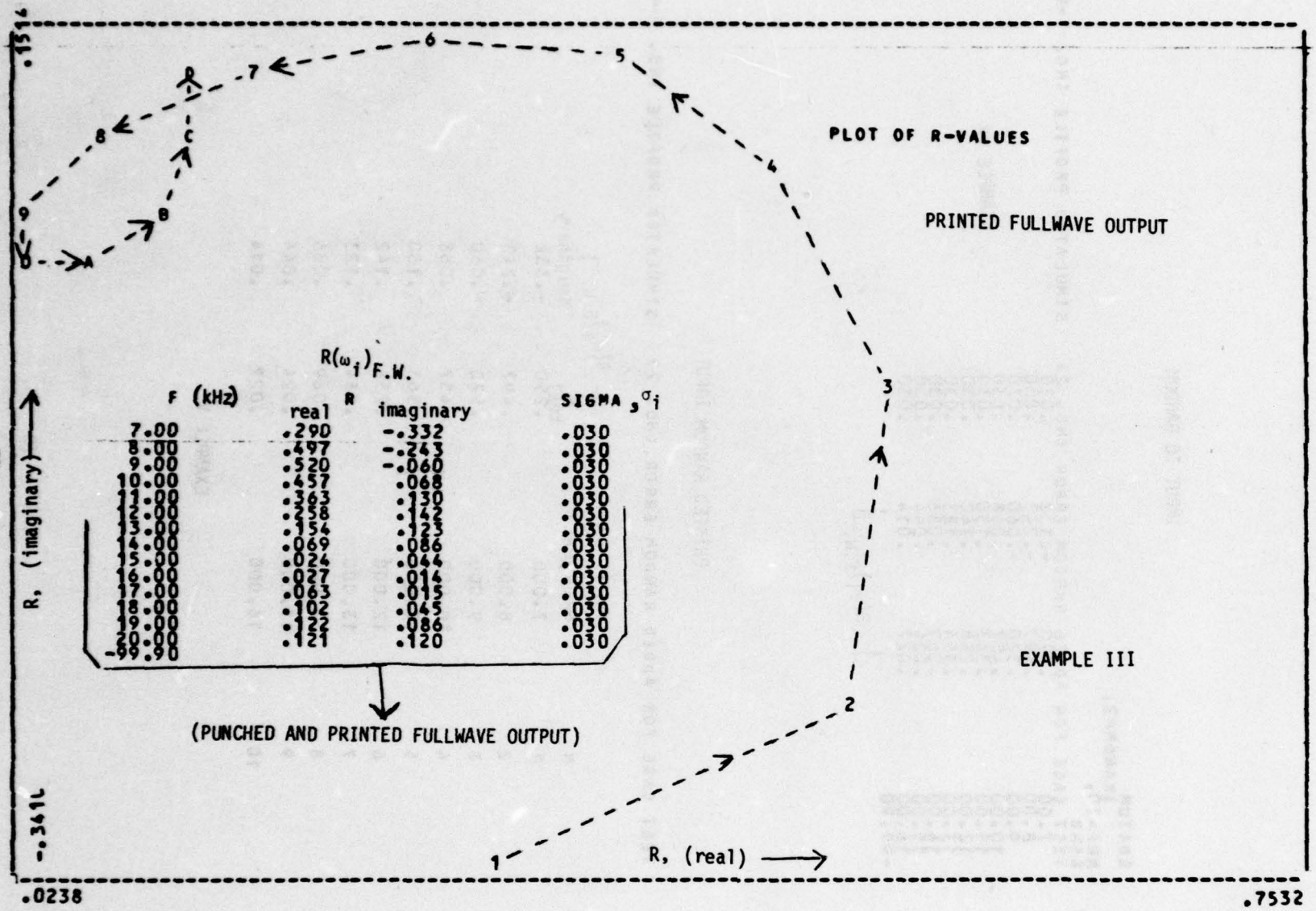
```
PROFILE 1
 SIMULATED DATA PROFILE (NO. 1-B)
100.00       2.10+003
 99.00       2.10+003
 98.00       2.10+003
 97.00       2.10+003
 96.00       2.10+003
 95.00       2.10+003
 94.00       2.10+003
 93.00       2.10+003
 92.00       2.05+003
 91.00       1.90+003
 90.00       1.52+003
 89.00       8.14+002
 88.00       5.11+002
 87.00       3.50+002
 86.00       3.08+002
 85.00       2.96+002
 84.00       2.93+002
 83.00       2.93+002
 82.00       2.75+002
 81.00       2.19+002
 80.00       1.71+002
 79.00       1.20+002
 78.00       8.70+001
 77.00       6.60+001
 76.00       5.13+001
 75.00       4.12+001
 74.00       3.97+001
 73.00       4.15+001
 72.00       4.72+001
 71.00       5.63+001
 70.00       6.84+001
 69.00       8.30+001
 68.00       9.60+001
 67.00       9.60+001
 66.00       8.98+001
 65.00       7.72+001
 64.00       6.60+001
 63.00       5.42+001
 62.00       4.60+001
 61.00       3.73+001
 60.00       2.82+001
 59.00       2.20+001
 58.00       1.70+001
 57.00       1.30+001
 56.00       1.00+001
 55.00       7.72+000
 54.00       5.65+000
 53.00       4.30+000
 52.00       3.23+000
 51.00       2.55+000
 50.00       1.93+000
```

EXAMPLE I

```
 &DATUM
 THETA=22.0,AZIM=90.0,CODIP=12.0,MAGFLD=5.3E-5,
 NUFLAG=1,
 FLOW=7.0,FINC=1.0,FHI=20.0,
 &END
WAVE
QUIT
```

HT	N
100.00	2.10+003
99.00	2.10+003
98.00	2.10+003
97.00	2.10+003
96.00	2.10+003
95.00	2.10+003
94.00	2.10+003
93.00	2.10+003
92.00	2.05+003
91.00	1.90+003
90.00	1.52+003
89.00	8.14+002
88.00	5.11+002
87.00	3.50+002
86.00	3.08+002
85.00	2.96+002
84.00	2.93+002
83.00	2.93+002
82.00	2.75+002
81.00	2.19+002
80.00	1.71+002
79.00	1.20+002
78.00	8.70+001
77.00	6.60+001
76.00	5.13+001
75.00	4.12+001
74.00	3.97+001
73.00	4.15+001
72.00	4.72+001
71.00	5.63+001
70.00	6.84+001
69.00	8.30+001
68.00	9.60+001
67.00	9.60+001
66.00	8.98+001
65.00	7.72+001
64.00	6.60+001
63.00	5.42+001
62.00	4.60+001
61.00	3.73+001
60.00	2.82+001
59.00	2.20+001
58.00	1.70+001
57.00	1.30+001
56.00	1.00+001
55.00	7.72+000
54.00	5.65+000
53.00	4.30+000
52.00	3.23+000
51.00	2.55+000
50.00	1.93+000





$R(\omega_i)_{F.W.}$

F (kHz)	R real	R imaginary	SIGMA, σ_i
7.00	.290	-.332	.030
8.00	.497	-.243	.030
9.00	.520	-.060	.030
10.00	.457	.068	.030
11.00	.363	.130	.030
12.00	.258	.142	.030
13.00	.154	.123	.030
14.00	.069	.086	.030
15.00	.024	.044	.030
16.00	.027	.014	.030
17.00	.063	.015	.030
18.00	.102	.045	.030
19.00	.122	.086	.030
20.00	.121	.120	.030
-99.90			

INPUT TO RANDOM

```
&DATUM
      IRANDM=2,
NRF=10,
&END
TEST CASE FOR ADDED RANDOM ERROR (NO. 2)   SIMULATED PROFILE (NO. 1-B)
   7.00          .290        -.332        .030
   8.00          .497        -.243        .030
   9.00          .520        -.060        .030
  10.00          .457         .068        .030
  11.00          .363         .130        .030
  12.00          .258         .142        .030
  13.00          .154         .123        .030
  14.00          .069         .086        .030
  15.00          .024         .044        .030
  16.00          .027         .014        .030
-99.90
                 |_ R(ω_i)_F.W._|
```

EXAMPLE IV

PRINTED RANDOM INPUT

TEST CASE FOR ADDED RANDOM ERROR (NO. 2) SIMULATED PROFILE (NO. 1-B)

N	INPUT DATA FREQUENCY	$\vdash R(\omega_i)_{F.W.} \dashv$ R real	imaginary
1	7.000	.290	-.332
2	8.000	.497	-.243
3	9.000	.520	-.060
4	10.000	.457	.068
5	11.000	.363	.130
6	12.000	.258	.142
7	13.000	.154	.123
8	14.000	.069	.086
9	15.000	.024	.044
10	16.000	.027	.014

EXAMPLE V

PRINTED RANDOM OUTPUT

TEST CASE FOR ADDED RANDOM ERROR (NO. 2) SIMULATED PROFILE (NO. 1-B)

GAUSSIAN RANDOM X DATA

I	IRANDM	XREAL($U_{i,q}$)	IRANDM	XIMAG($U_{i,q}$)
1	1331975123	-3.233	1192245589	1.687
2	1582285849	.281	731406625	-1.123
3	875543089	-1.896	527354449	-1.523
4	1284630161	-.499	916625169	-.049
5	994699281	.051	1634101777	-.549
6	1451175441	-.445	1752708625	1.271
7	118048273	.708	763979281	-1.057
8	830842385	.587	1744184849	.632
9	583476753	.045	1984505361	.145
10	487400977	-.858	1913988625	-.770

EXAMPLE VI

PRINTED RANDOM OUTPUT EXAMPLE VII

TEST CASE FOR ADDED RANDOM ERROR (NO. 2) SIMULATED PROFILE (NO. 1-B)

GAUSSIAN RANDOM R DATA

N	FREQUENCY	R $R(\omega_i)_{F.W.}$		RR $\left(R(\omega_i)_{\varepsilon}\right)$		SIGMA
1	7.000	.290	-.332	.193	-.281	.030
2	8.000	.497	-.243	.505	-.277	.030
3	9.000	.520	-.060	.463	-.106	.030
4	10.000	.457	.068	.442	.067	.030
5	11.000	.363	.130	.365	.114	.030
6	12.000	.258	.142	.245	.180	.030
7	13.000	.154	.123	.175	.091	.030
8	14.000	.069	.086	.087	.105	.030
9	15.000	.024	.044	.025	.048	.030
10	16.000	.027	.014	.001	-.009	.030

PRINTED AND PUNCHED RANDOM OUTPUT EXAMPLE VIII

SIMULATED DATA PROFILE (NO. 1-B) ((FOR THULE))
*** ADDED RANDOM ERROR *** (SET NO. 2)

7.00	.193	-.281	.030
8.00	.505	-.277	.030
9.00	.463	-.106	.030
10.00	.442	.067	.030
11.00	.365	.114	.030
12.00	.245	.180	.030
13.00	.175	.091	.030
14.00	.087	.105	.030
15.00	.025	.048	.030
16.00	.001	-.009	.030
-99.90			

$R(\omega_i)_{\varepsilon}$

TEST CASE FOR ADDED RANDOM ERROR (NO. 3) SIMULATED PROFILE (NO. 1-B)

PRINTED RANDON INPUT EXAMPLE IX

```
$DATUM
      IRANDM=3,
NRF=10,
$END
TEST CASE FOR ADDED RANDOM ERROR (NO. 3)  SIMULATED PROFILE (NO. 1-B)
   7.000          .290        -.332        .030
   8.000          .497        -.243        .030
   9.000          .520        -.060        .030
  10.000          .457         .068        .030
  11.000          .363         .130        .030
  12.000          .258         .142        .030
  13.000          .154         .123        .030
  14.000          .069         .086        .030
  15.000          .024         .044        .030
  16.000          .027         .014        .030
-99.000
```

$R(\omega_i)_{F.W.}$

PRINTED RANDOM OUTPUT EXAMPLE X

TEST CASE FOR ADDED RANDOM ERROR (NO. 3) SIMULATED PROFILE (NO. 1-B)

GAUSSIAN RANDOM X DATA

I	IRANDM	XREAL	IRANDM	XIMAG
1	786302773	-3.722	1805951365	.650
2	1225796925	-.041	408665393	.178
3	1657140705	-1.673	804024977	.026
4	1069125	1.240	404625425	-1.096
5	1795738129	.388	980938257	-1.499
6	458339857	.024	629949969	.374
7	1110402577	.470	370255377	.782
8	1556[illegible]465	-1.199	5195113617	-.052
9	1755415793	.075	902637073	-.748
10	194323985	-1.051	1042621969	.039

PRINTED RANDOM OUTPUT

TEST CASE FOR ADDED RANDOM ERROR (NO. 3) SIMULATED PROFILE (NO. 1-B)

GAUSSIAN RANDOM R DATA

N	FREQUENCY	$R(\omega_i)_{F.W.}$		$RR_{(R(\omega_i)_\varepsilon)}$		SIGMA
1	7.000	.290	-.332	.178	-.313	.030
2	8.000	.497	-.243	.496	-.238	.030
3	9.000	.520	-.060	.470	-.059	.030
4	10.000	.457	.068	.494	.035	.030
5	11.000	.363	.130	.375	.085	.030
6	12.000	.258	.142	.259	.153	.030
7	13.000	.154	.123	.168	.146	.030
8	14.000	.069	.086	.033	.084	.030
9	15.000	.024	.044	.026	.022	.030
10	16.000	.027	.014	-.005	.015	.030

PRINTED AND PUNCHED RANDOM OUTPUT

TEST CASE FOR ADDED RANDOM ERROR (NO. 3) SIMULATED PROFILE (NO. 1-B)

7.00	.178	-.313	.030
8.00	.496	-.238	.030
9.00	.470	-.059	.030
10.00	.494	.035	.030
11.00	.375	.085	.030
12.00	.259	.153	.030
13.00	.168	.146	.030
14.00	.033	.084	.030
15.00	.026	.022	.030
16.00	-.005	.015	.030
-99.90			

$R(\omega_i)_\varepsilon$

EXAMPLE XI

```
SIMULATED DATA PROFILE (NO. 1-B)     ((FOR THULE))
     *** ADDED RANDOM ERROR ***     (SET NO. 2)
   7.00          .193        -.281        .030
   8.00          .505        -.277        .030
   9.00          .463        -.106        .030
  10.00          .442         .067        .030
  11.00          .365         .114        .030
  12.00          .245         .180        .030
  13.00          .175         .091        .030
  14.00          .087         .105        .030
  15.00          .025         .048        .030
  16.00          .001        -.009        .030
-99.90
WAVE
&FULLWV
THETA=22.0,AZIM=90.0,CODIP=12.0,MAGFLD=5.3E-5,
&END
SIMULATED DATA PROFILE (NO. 1-B)
100.          2.10E+03
99.0          2.10E+03
98.0          2.10E+03
97.0          2.10E+03
96.0          2.10E+03
95.0          2.10E+03
94.0          2.10E+03
93.0          2.10E+03
92.0          2.05E+03
91.0          1.90E+03                          INVERT INPUT
90.0          1.52E+03
89.0          8.14E+02                          EXAMPLE XII
88.0          5.11E+02
87.0          3.50E+02
86.0          3.08E+02
85.0          2.96E+02
84.0          2.93E+02
83.0          2.93E+02
82.0          2.75E+02
81.0          2.19E+02
80.0          1.71E+02
79.0          1.20E+02
78.0          8.70E+01
77.0          6.60E+01
76.0          5.13E+01
75.0          4.12E+01
74.0          3.97E+01
73.0          4.15E+01
72.0          4.72E+01
71.0          5.63E+01
70.0          6.84E+01
69.0          8.30E+01
68.0          9.60E+01
67.0          9.60E+01
66.0          8.98E+01
65.0          7.72E+01
64.0          6.60E+01

63.0          5.42E+01
62.0          4.60E+01
61.0          3.73E+01                    NAME
60.0          2.82E+01                     &DATUM
59.0          2.20E+01                     HTSTOP=100.0,50.0,
58.0          1.70E+01                     STOPS=700.0,3.0,
57.0          1.30E+01                     NUFLAG=1,
56.0          1.00E+01                     DOPT=0.4,
55.0          7.72E+00                     DELTAL=0.0625,
54.0          5.65E+00                     ROTATE=+90.0,
53.0          4.30E+00                     MIN=9,
52.0          3.23E+00                     &END
51.0          2.55E+00
50.0          1.93E+00
 -99.0
```

```
 SIMULATED DATA PROFILE (NO. 1-B)     ((FOR THULE))
     *** ADDED RANDON ERROR ***     (SET NO. 2)
-------------------------------------------------
   7.00          .193        -.281        .030
   8.00          .505        -.277        .030
   9.00          .463        -.106        .030
  10.00          .442         .067        .030
  11.00          .365         .114        .030
  12.00          .245         .180        .030
  13.00          .175         .091        .030
  14.00          .087         .105        .030
  15.00          .025         .048        .030
  16.00          .001        -.009        .030
 -99.90          .000         .000        .000
FULL WAVE IN PUT
                 L-R(ω_i)_ε --J
WAVE
  &DATUM
  THETA =    22.000 DEGREES AZIM =    90.000 DEGREES
 IPRINT =   0 IPROFP =   1 IRPLOT =   1
  CODIP =    12.000 DEGREES MAGFLD =  5.300-005

  &END


  SIMULATED DATA PROFILE (NO. 1-B)
 NAME
  &DATUM
  HTSTOP(1) =   100.000 HTSTOP(2) =    50.000

  STOPS(1) =  7.000+002 STOPS(2) =  3.000+000
                  IPRNTA =   0 IPRNTE =   0 NUFLAG =   1

   MIN =      9 DOPT =          .400 DELTAH =        1.000
   ROTATE =    90.000 ALAMDA =          .000 DELTAL =          .0625
   NUFLAG =   1 COEFNU =  4.303+011 EXPNU =       -.1622
   &END
```

PRINTOUT OF INVERT INPUT EXAMPLE XIII

IN DGELB, IER = 0

β - values from equation (34)

-.25267	.31921	-.08406	.02214	-.00583
-.25267	.31921	-.08406	.02214	-.00583
-.25107	.31560	-.08152	.02147	-.00565
-.24629	.30476	-.07387	.01945	-.00512
-.23831	.28670	-.06114	.01610	-.00424
-.22713	.26141	-.04331	.01140	-.00300
-.21277	.22890	-.02038	.00537	-.00141
-.19521	.18916	.00764	-.00201	.00053
-.17446	.14220	.04076	-.01073	.00283

→ etc

↓ etc

CLINEQ ERR = .000

Computed values of σ_{ii} from equation (45)

3.33+001	.00	.00	.00	.00
.00	.00	.00	.00	.00
3.33+001	6.58+000	3.10+000	-1.22+000	5.22+000
-5.61-008	4.88+000	-9.45+000	4.28+000	-2.94+000
3.33+001	7.88+000	7.64-001	-5.99-001	2.57+000
2.87-008	5.84+000	-2.33+000	2.11+000	-1.45+000
3.33+001	7.88+000	1.14-008	-1.52+000	5.22+000
6.15-008	5.84+000	1.72-009	5.34+000	-2.94+000

→ etc

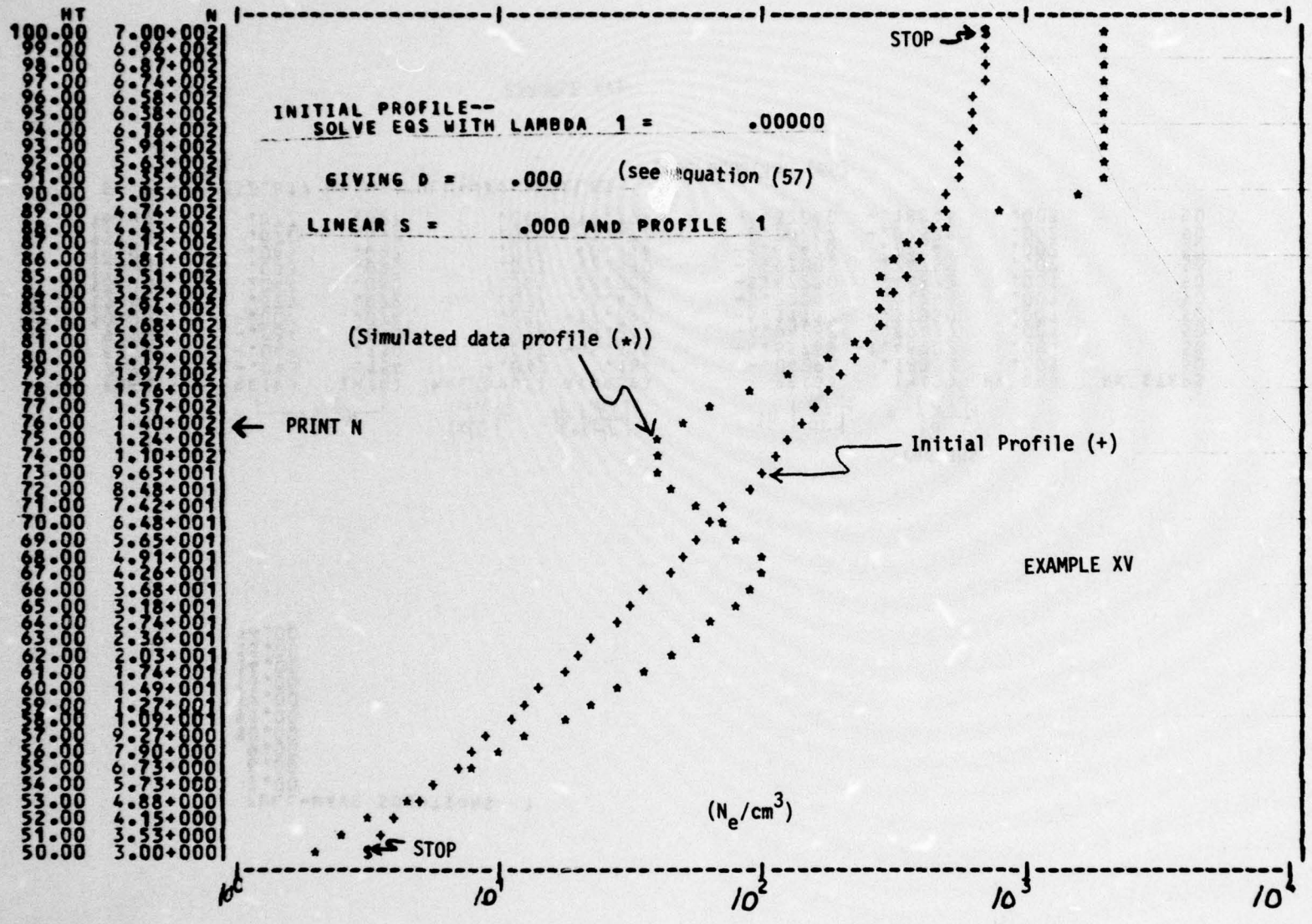
↓ etc

DATA VALUES-- $R(\omega_i)_\varepsilon$ $\left|\frac{dR_\varepsilon}{d\omega}\right|_i$ $\phi\left\{\frac{dR}{d\omega}\right\}_i$ $\ln\left|\frac{dR_\varepsilon}{d\omega}\right|_i$ $\phi\left\{\frac{dR_\varepsilon}{d\omega}\right\}_i$ radians

FREQ	RE(R)	IM(R)	MAG(DRDF)	A(DRDF)	RE(G)	IM(G)
7.00	.193	-.281	.193	-.281	.19300	-.28100
8.00	.505	-.277	.029	36.552	-3.52534	.63796
9.00	.463	-.106	.045	108.173	-3.11172	1.88797
10.00	.442	.067	.020	105.858	-3.91548	1.84757
11.00	.365	.114	.027	150.606	-3.61752	2.62857
12.00	.325	.160	.019	192.487	-3.94154	3.35653
13.00	.175	.091	.018	214.276	-4.02412	3.73982
14.00	.087	.105	.017	181.259	-4.08574	3.16357
15.00	.025	-.048	.016	241.435	-4.10440	4.21384
16.00	.001	-.009	.010	251.639	-4.56526	4.39193

END OF INITILIZATION

EXAMPLE XIV PRINTED INVERT OUTPUT


HT N
100.00 7.00+002
99.00 6.96+002
98.00 6.87+002
97.00 6.74+002
96.00 6.58+002
95.00 6.38+002
94.00 6.16+002
93.00 5.91+002
92.00 5.63+002
91.00 5.35+002
90.00 5.05+002
89.00 4.74+002
88.00 4.43+002
87.00 4.12+002
86.00 3.81+002
85.00 3.51+002
84.00 3.22+002
83.00 2.94+002
82.00 2.68+002
81.00 2.43+002
80.00 2.19+002
79.00 1.97+002
78.00 1.76+002
77.00 1.57+002
76.00 1.40+002
75.00 1.24+002
74.00 1.10+002
73.00 9.65+001
72.00 8.48+001
71.00 7.42+001
70.00 6.48+001
69.00 5.65+001
68.00 4.91+001
67.00 4.26+001
66.00 3.68+001
65.00 3.18+001
64.00 2.74+001
63.00 2.36+001
62.00 2.03+001
61.00 1.74+001
60.00 1.49+001
59.00 1.27+001
58.00 1.09+001
57.00 9.27+000
56.00 7.90+000
55.00 6.73+000
54.00 5.73+000
53.00 4.88+000
52.00 4.15+000
51.00 3.53+000
50.00 3.00+000
INITIAL PROFILE--
SOLVE EQS WITH LAMBDA 1 = .00000
GIVING D = .000 (see equation (57)
LINEAR S = .000 AND PROFILE 1
STOP
(Simulated data profile (*))
PRINT N
Initial Profile (+)
EXAMPLE XV
(N_e/cm^3)
STOP
10^0
10^1
10^2
10^3
10^4

FULL-WAVE SOLUTIONS 1

7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
16.00

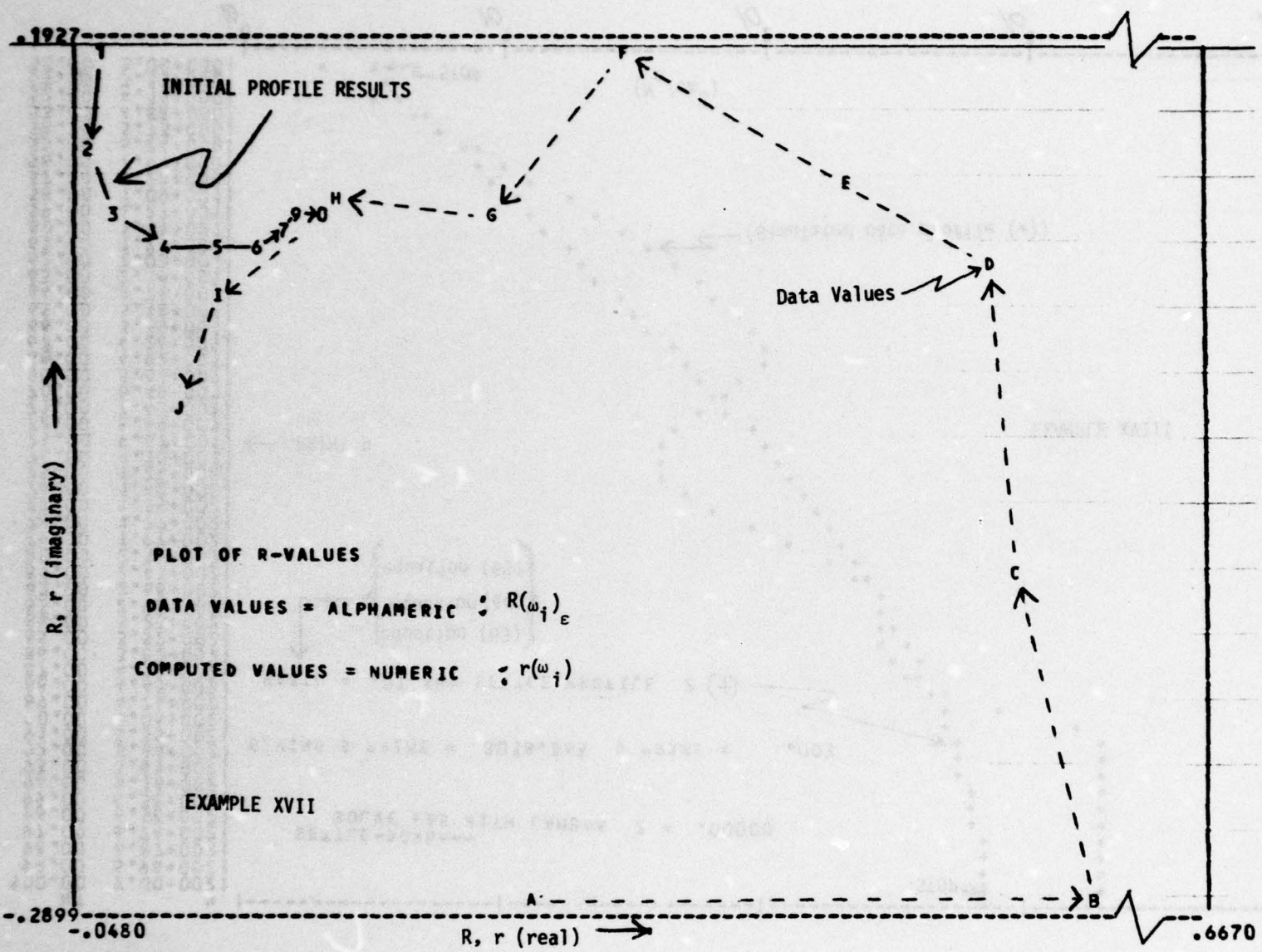
radians

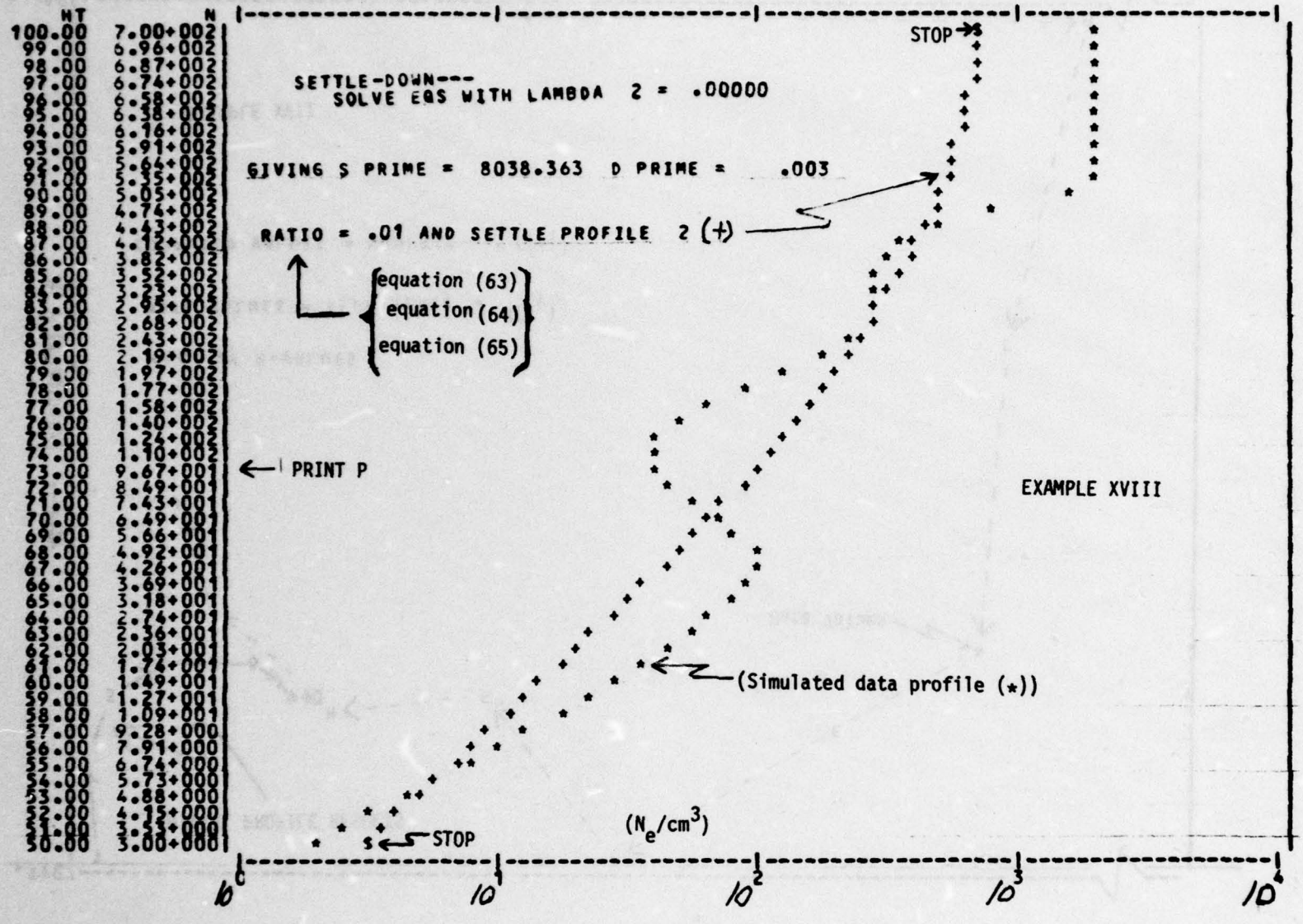
FREQ	$r(\omega_i)$ RE(R)	$r(\omega_i)$ IM(R)	$\left\|\frac{dr}{d\omega}\right\|_i$ MAG(DRDF)	$\phi\left\{\frac{dr}{d\omega}\right\}_i$ A(DRDF)	$\ln\left\|\frac{dr}{d\omega}\right\|_i$ RE(G)	$\phi\left\{\frac{d}{d\omega}\right\}_i$ IM(G)	RK ERR	RK STEPS
7.00	-.043	.184	-.043	.184	-.04296	.18373	.001	50
8.00	-.048	.130	.047	-85.315	-4.65769	-1.48902	.001	50
9.00	-.034	.093	.036	-51.267	-4.94268	-.89477	.001	50
10.00	-.007	.074	.030	-17.790	-5.11117	-.31050	.001	50
11.00	.021	.072	.027	8.477	-5.22012	.14795	.001	50
12.00	.044	.080	.021	28.002	-5.47270	.48872	.001	50
13.00	.058	.090	.013	42.830	-5.93003	.74753	.002	50
14.00	.065	.097	.006	40.994	-6.65478	.71548	.002	50
15.00	.070	.098	.006	-1.477	-6.70787	-.02578	.002	50
16.00	.077	.097	.009	-10.799	-6.37040	-.18848	.003	50

S 1 = 8032.817 USING FULL-WAVE VALUES

(see equation (66)

EXAMPLE XVI


.1927
INITIAL PROFILE RESULTS
1
2
3
4
5
6
7
8
9
10
F
E
D
C
B
A
G
H
I
J
Data Values
PLOT OF R-VALUES
DATA VALUES = ALPHAMERIC : $R(\omega_i)_\varepsilon$
COMPUTED VALUES = NUMERIC : $r(\omega_i)$
EXAMPLE XVII
R, r (imaginary)
-.2899
-.0480
R, r (real)
.6670


HT N
100.00 7.00+002
99.00 6.96+002
98.00 6.87+002
97.00 6.74+002
96.00 6.58+002
95.00 6.38+002
94.00 6.16+002
93.00 5.91+002
92.00 5.64+002
91.00 5.35+002
90.00 5.05+002
89.00 4.74+002
88.00 4.43+002
87.00 4.12+002
86.00 3.82+002
85.00 3.52+002
84.00 3.23+002
83.00 2.95+002
82.00 2.68+002
81.00 2.43+002
80.00 2.19+002
79.00 1.97+002
78.00 1.77+002
77.00 1.58+002
76.00 1.40+002
75.00 1.24+002
74.00 1.10+002
73.00 9.67+001
72.00 8.49+001
71.00 7.43+001
70.00 6.49+001
69.00 5.66+001
68.00 4.92+001
67.00 4.26+001
66.00 3.69+001
65.00 3.18+001
64.00 2.74+001
63.00 2.36+001
62.00 2.03+001
61.00 1.74+001
60.00 1.49+001
59.00 1.27+001
58.00 1.09+001
57.00 9.28+000
56.00 7.91+000
55.00 6.74+000
54.00 5.73+000
53.00 4.88+000
52.00 4.15+000
51.00 3.55+000
50.00 3.00+000
SETTLE-DOWN---
SOLVE EQS WITH LAMBDA 2 = .00000
GIVING S PRIME = 8038.363 D PRIME = .003
RATIO = .01 AND SETTLE PROFILE 2 (+)
equation (63)
equation (64)
equation (65)
PRINT P
STOP
STOP
EXAMPLE XVIII
(Simulated data profile (*))
(N_e/cm^3)
10^0
10^1
10^2
10^3
10^4

NEW LAMBDA---

(a) DELTA LAMBDA 3 = .08839

(b) SOLVE EQS WITH LAMBDA 3 = .08839

(c) GIVING D = .241 LINEAR S = 6874.953 AND PROFILE 3

(d) FULL-WAVE SOLUTIONS 3

(e)

FREQ	RE(R)	IM(R)	MAG(DRDF)	A(DRDF)	RE(G)	IM(G)
7.00	-.056	.159	-.056	.159	-.05577	.15869
8.00	-.048	.108	.046	-69.578	-4.68869	-1.21437
9.00	-.023	.076	.036	-34.336	-4.94305	-.59928
10.00	.007	.067	.029	.094	-5.13483	.00163
11.00	.034	.074	.026	27.527	-5.25919	.48044
12.00	.052	.088	.020	46.226	-5.53450	.80679
13.00	.066	.100	.012	55.184	-6.06836	.96315
14.00	.067	.106	.006	28.122	-6.81095	.49081
15.00	.073	.106	.008	-6.951	-6.40821	-.12132
16.00	.083	.105	.011	-6.682	-6.12248	-.11663

(f) S 3 = 6796.962 USING FULL-WAVE VALUES

(g) PLOT OF R-VALUES

(h) SETTLE-DOWN---
SOLVE EQS WITH LAMBDA 4 = .08839

(i) GIVING S PRIME = 6363.873 D PRIME = .207

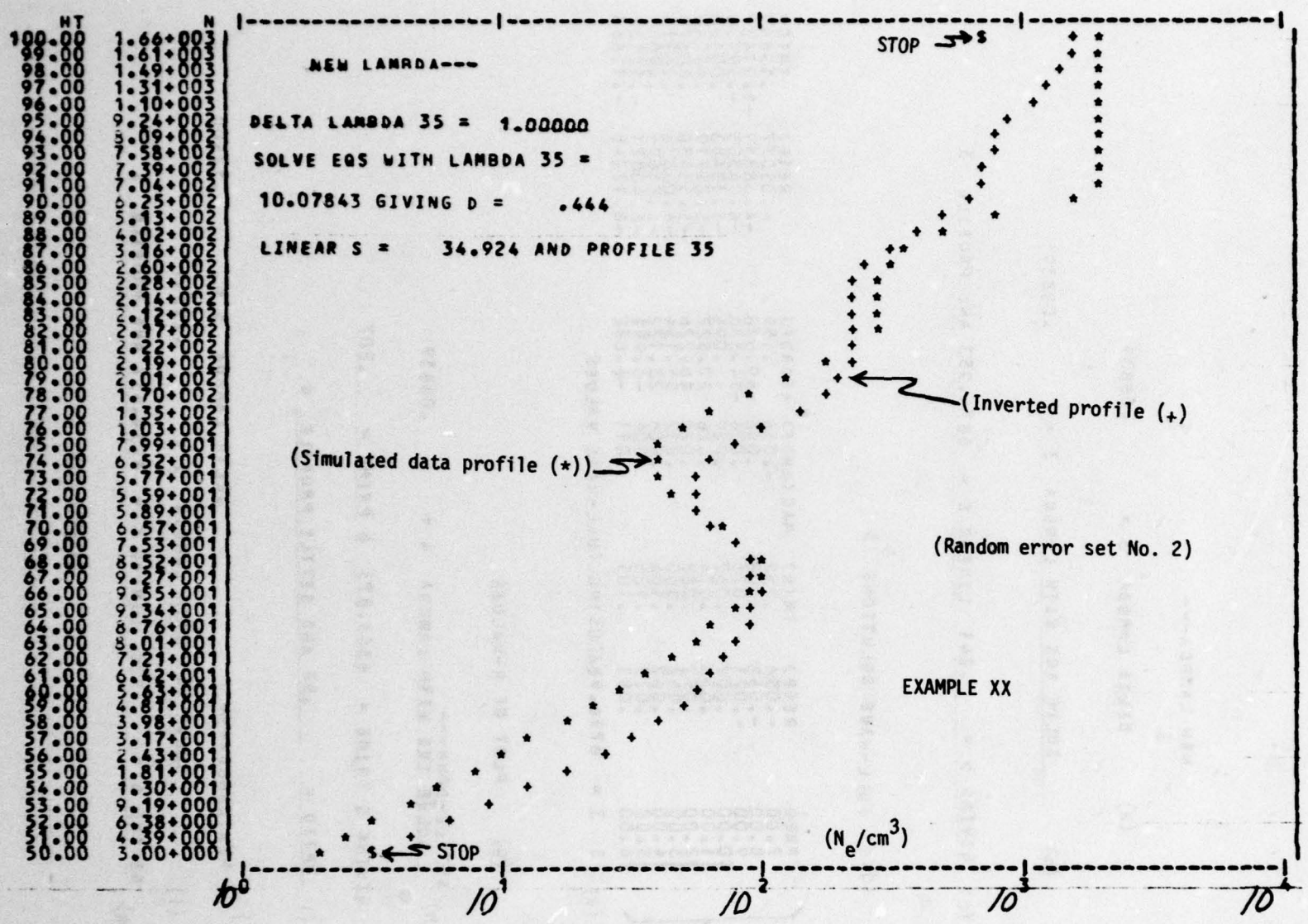
(j) RATIO = .52 AND SETTLE PROFILE 4

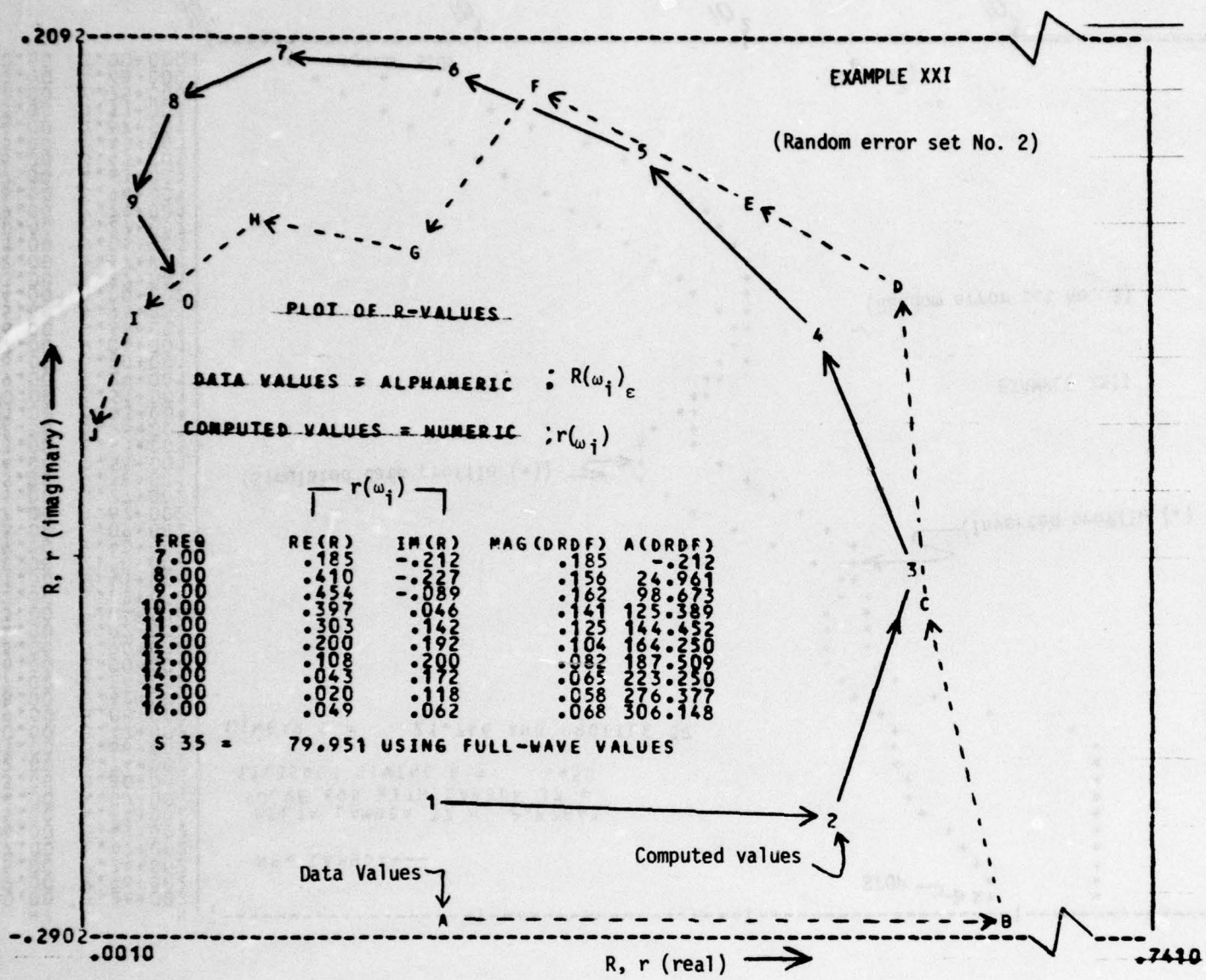
(k) NEW LAMBDA--- DELTA LAMBDA 5 = .12500

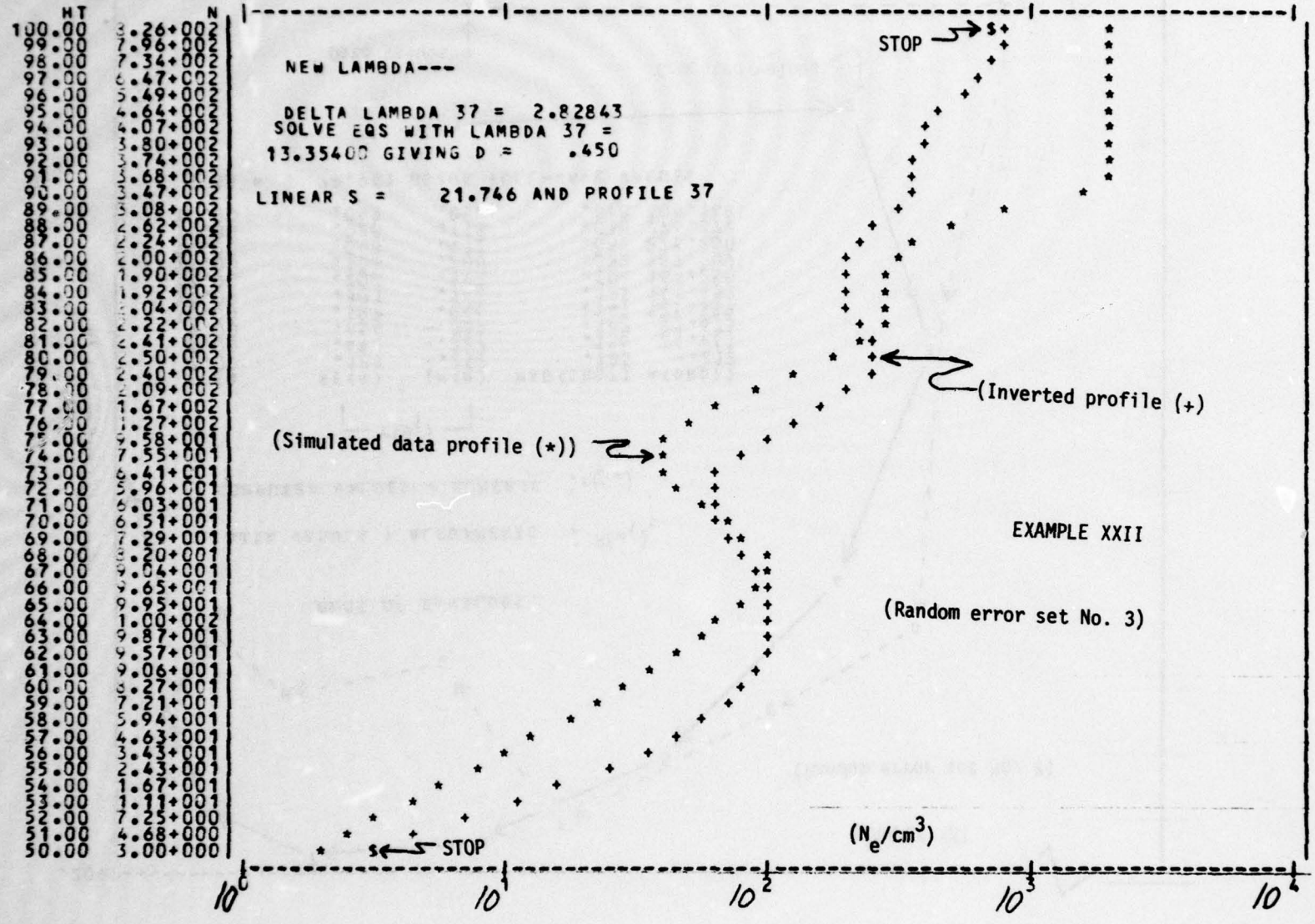
(l) SOLVE EQS WITH LAMBDA 5 = .21339

(m) GIVING D = .728 LINEAR S = 4239.557 AND PROFILE 5

EXAMPLE XIX


HT N
100.00 1.66+003
99.00 1.61+003
98.00 1.49+003
97.00 1.31+003
96.00 1.10+003
95.00 9.24+002
94.00 8.09+002
93.00 7.58+002
92.00 7.39+002
91.00 7.04+002
90.00 6.25+002
89.00 5.13+002
88.00 4.02+002
87.00 3.16+002
86.00 2.60+002
85.00 2.28+002
84.00 2.14+002
83.00 2.12+002
82.00 2.17+002
81.00 2.22+002
80.00 2.19+002
79.00 2.01+002
78.00 1.70+002
77.00 1.35+002
76.00 1.03+002
75.00 7.99+001
74.00 6.52+001
73.00 5.77+001
72.00 5.59+001
71.00 5.89+001
70.00 6.57+001
69.00 7.53+001
68.00 8.52+001
67.00 9.27+001
66.00 9.55+001
65.00 9.34+001
64.00 8.76+001
63.00 8.01+001
62.00 7.21+001
61.00 6.42+001
60.00 5.65+001
59.00 4.81+001
58.00 3.98+001
57.00 3.17+001
56.00 2.43+001
55.00 1.81+001
54.00 1.30+001
53.00 9.19+000
52.00 6.38+000
51.00 4.39+000
50.00 3.00+000
NEW LAMBDA---
DELTA LAMBDA 35 = 1.00000
SOLVE EQS WITH LAMBDA 35 =
10.07843 GIVING D = .444
LINEAR S = 34.924 AND PROFILE 35
STOP
(Inverted profile (+)
(Simulated data profile (*))
(Random error set No. 2)
EXAMPLE XX
STOP
(N_e/cm^3)
10^0
10^1
10^2
10^3
10^4

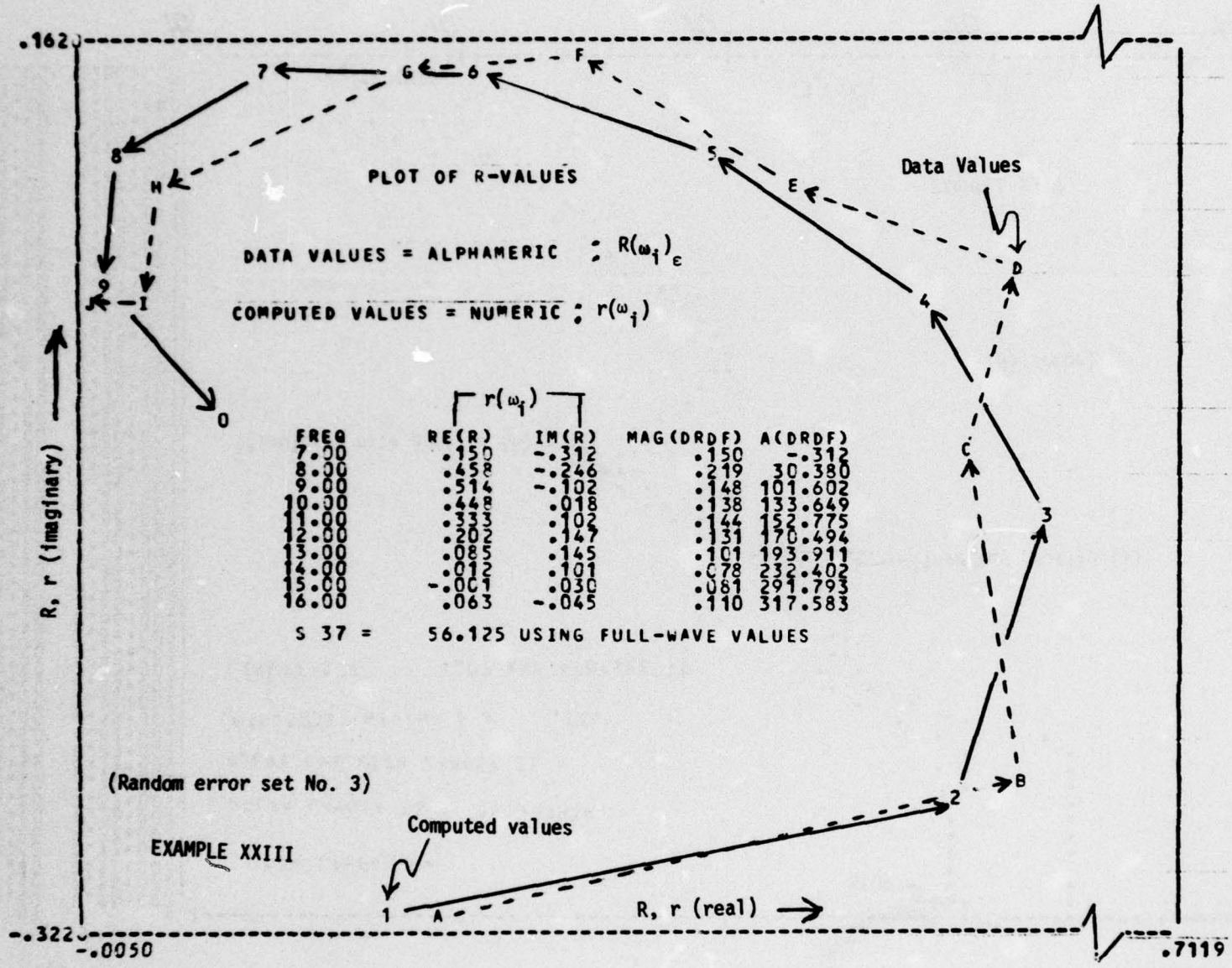

EXAMPLE XXI
(Random error set No. 2)
PLOT OF R-VALUES
DATA VALUES = ALPHAMERIC ; $R(\omega_i)_\varepsilon$
COMPUTED VALUES = NUMERIC ; $r(\omega_i)$
$r(\omega_i)$
FREQ RE(R) IM(R) MAG(DRDF) A(DRDF)
7.00 .185 -.212 .185 -.212
8.00 .410 -.227 .156 24.961
9.00 .454 -.089 .162 98.673
10.00 .397 .046 .141 125.389
11.00 .303 .142 .125 144.452
12.00 .200 .192 .104 164.250
13.00 .108 .200 .082 187.509
14.00 .043 .172 .065 223.250
15.00 .020 .118 .058 276.377
16.00 .049 .062 .068 306.148
S 35 = 79.951 USING FULL-WAVE VALUES
Computed values
Data Values
R, r (imaginary)
R, r (real)
.2092
-.2902
.0010
.7410


HT N
100.00 3.26+002
99.00 7.96+002
98.00 7.34+002
97.00 6.47+002
96.00 5.49+002
95.00 4.64+002
94.00 4.07+002
93.00 3.80+002
92.00 3.74+002
91.00 3.68+002
90.00 3.47+002
89.00 3.08+002
88.00 2.62+002
87.00 2.24+002
86.00 2.00+002
85.00 1.90+002
84.00 1.92+002
83.00 2.04+002
82.00 2.22+002
81.00 2.41+002
80.00 2.50+002
79.00 2.40+002
78.00 2.09+002
77.00 1.67+002
76.00 1.27+002
75.00 9.58+001
74.00 7.55+001
73.00 6.41+001
72.00 5.96+001
71.00 6.03+001
70.00 6.51+001
69.00 7.29+001
68.00 8.20+001
67.00 9.04+001
66.00 9.65+001
65.00 9.95+001
64.00 1.00+002
63.00 9.87+001
62.00 9.57+001
61.00 9.06+001
60.00 8.27+001
59.00 7.21+001
58.00 5.94+001
57.00 4.63+001
56.00 3.43+001
55.00 2.43+001
54.00 1.67+001
53.00 1.11+001
52.00 7.25+000
51.00 4.68+000
50.00 3.00+000
NEW LAMBDA---
DELTA LAMBDA 37 = 2.82843
SOLVE EQS WITH LAMBDA 37 =
13.35400 GIVING D = .450
LINEAR S = 21.746 AND PROFILE 37
STOP
(Inverted profile (+)
(Simulated data profile (*))
EXAMPLE XXII
(Random error set No. 3)
STOP
(N_e/cm^3)
10^0
10^1
10^2
10^3
10^4

AD-A050 256 NAVAL OCEAN SYSTEMS CENTER SAN DIEGO CA F/G 20/14
'INVERT', A COMPUTER PROGRAM FOR OBTAINING D-REGION ELECTRON DE--ETC(U)
NOV 77 D G MORFITT, C H SHELLMAN DNA-MIPR-77-521
UNCLASSIFIED NOSC-IR-782 NL

2 OF 2
AD A050256

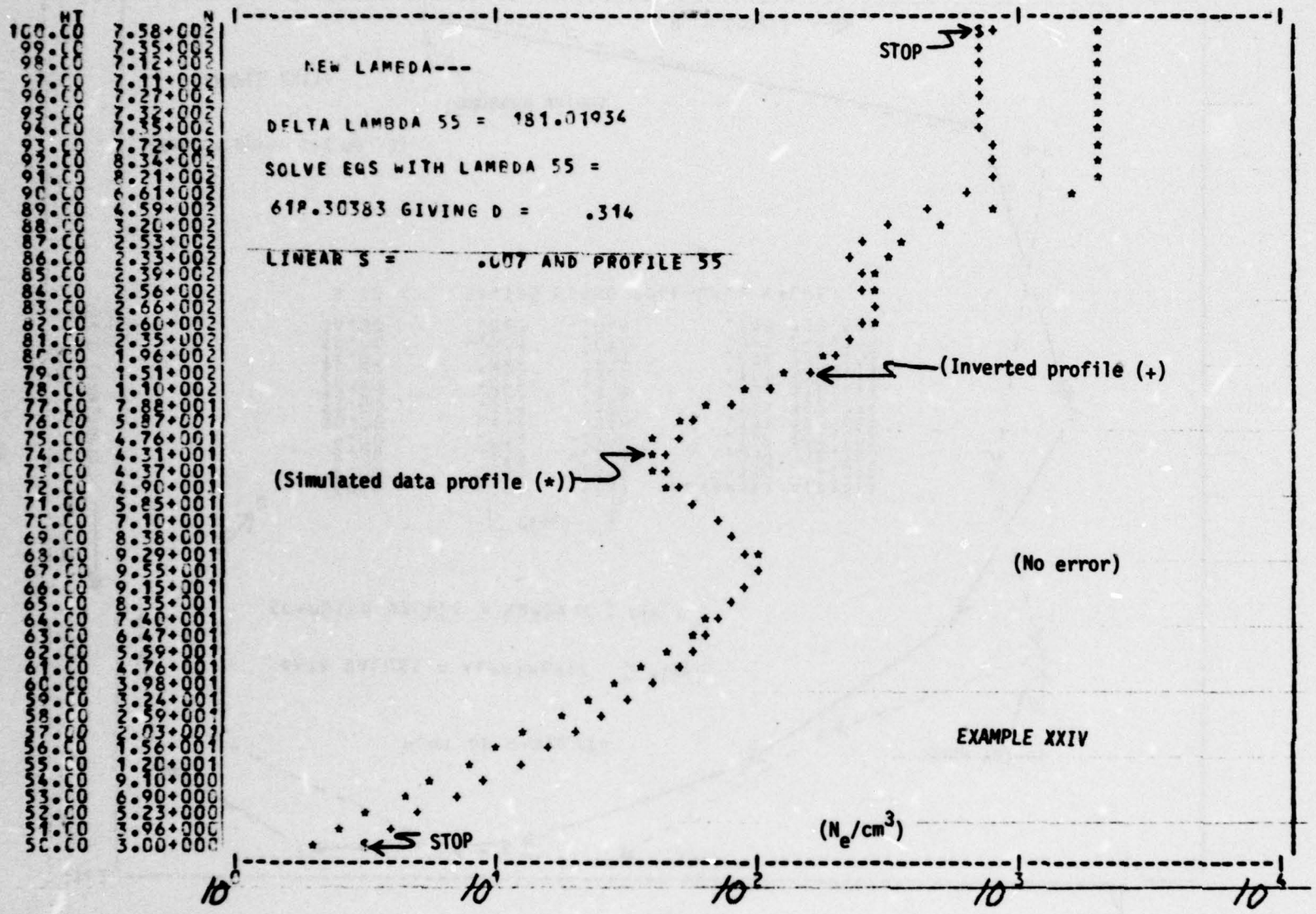
END
DATE FILMED
3-78
DDC

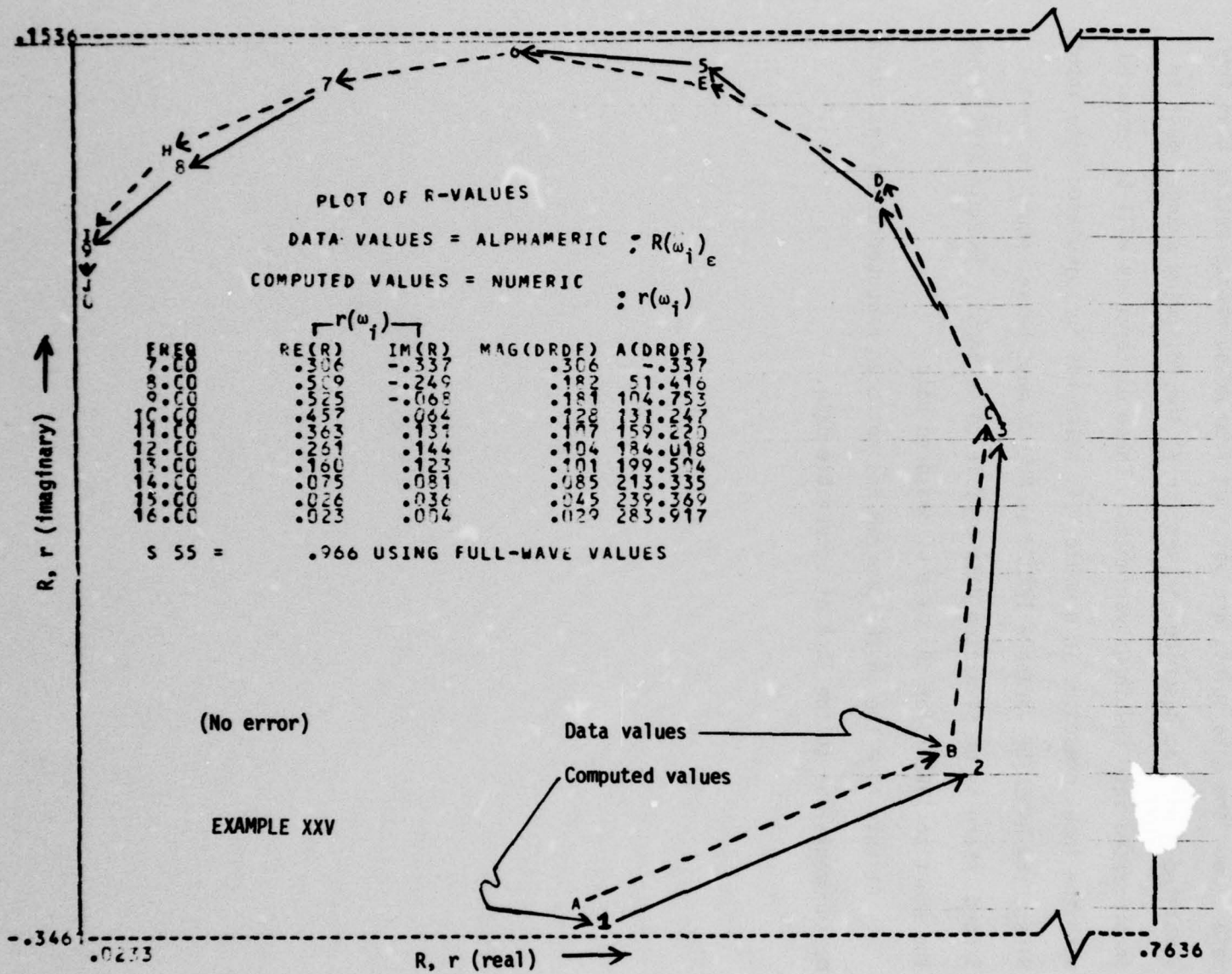


$r(\omega_i)$

FREQ	RE(R)	IM(R)	MAG(DRDF)	A(DRDF)
7.00	.150	-.312	.150	-.312
8.00	.458	-.246	.219	30.380
9.00	.514	-.102	.148	101.602
10.00	.448	.018	.138	133.649
11.00	.333	.102	.144	152.775
12.00	.202	.147	.131	170.494
13.00	.085	.145	.101	193.911
14.00	.012	.101	.078	232.402
15.00	-.001	.030	.081	291.793
16.00	.063	-.045	.110	317.583

S 37 = 56.125 USING FULL-WAVE VALUES


HT N
100.00 7.58+002
99.00 7.35+002
98.00 7.12+002
97.00 7.11+002
96.00 7.27+002
95.00 7.32+002
94.00 7.55+002
93.00 7.72+002
92.00 8.34+002
91.00 8.21+002
90.00 6.61+002
89.00 4.59+002
88.00 3.20+002
87.00 2.53+002
86.00 2.33+002
85.00 2.39+002
84.00 2.56+002
83.00 2.66+002
82.00 2.60+002
81.00 2.35+002
80.00 1.96+002
79.00 1.51+002
78.00 1.10+002
77.00 7.88+001
76.00 5.87+001
75.00 4.76+001
74.00 4.31+001
73.00 4.37+001
72.00 4.90+001
71.00 5.85+001
70.00 7.10+001
69.00 8.38+001
68.00 9.29+001
67.00 9.55+001
66.00 9.15+001
65.00 8.35+001
64.00 7.40+001
63.00 6.47+001
62.00 5.59+001
61.00 4.76+001
60.00 3.98+001
59.00 3.24+001
58.00 2.59+001
57.00 2.03+001
56.00 1.56+001
55.00 1.20+001
54.00 9.10+000
53.00 6.90+000
52.00 5.23+000
51.00 3.96+000
50.00 3.00+000
NEW LAMBDA---
DELTA LAMBDA 55 = 181.01934
SOLVE EQS WITH LAMBDA 55 =
618.30383 GIVING D = .314
LINEAR S = .007 AND PROFILE 55
STOP
(Inverted profile (+)
(Simulated data profile (*))
(No error)
EXAMPLE XXIV
STOP
(N_e/cm^3)
10^0
10^1
10^2
10^3
10^4


.1536
-.3461
.0273
.7636
R, r (real)
R, r (imaginary)
PLOT OF R-VALUES
DATA VALUES = ALPHAMERIC : $R(\omega_i)_\epsilon$
COMPUTED VALUES = NUMERIC : $r(\omega_i)$
$r(\omega_i)$
FREQ RE(R) IM(R) MAG(DRDF) A(DRDF)
7.00 .306 -.337 .306 -.337
8.00 .509 -.249 .182 51.416
9.00 .525 -.068 .181 104.753
10.00 .457 .064 .128 131.247
11.00 .363 .139 .107 159.220
12.00 .261 .144 .104 184.018
13.00 .160 .123 .101 199.504
14.00 .075 .081 .085 213.335
15.00 .026 .036 .045 239.369
16.00 .023 .004 .029 283.917
S 55 = .966 USING FULL-WAVE VALUES
(No error)
Data values
Computed values
EXAMPLE XXV
A
B
C
D
E
F
G
H
I
J
1
2
3
4
5
6
7
8
9

VI. COMMENTS ON EXECUTION OF INVERT FOR MEASURED DATA

For execution of the INVERT computer program using real experimental data (i.e., the data points are obtained from measurements rather than from simulated values as described in section V) the following changes must be incorporated into the input variables. These differences will be compared with the input presented in example XII, which was for simulated data input. First, the NAMELIST variable IPROFP in FULLWV must be set equal to zero. Second, there is no simulated data profile to be input. Example XXVI shows the input to INVERT for the case of measured data.

At this time the INVERT program has not been executed using real data measurements due to the lack of acceptable data.

```
INFORMATION CARD NO. 1 FOR MEASURED DATA
INFORMATION CARD NO. 2 FOR MEASURED DATA
   7.00            .290          -.332          .030
   8.00            .497          -.243          .030
   9.00            .520          -.060          .030
  10.00            .457           .068          .030
  11.00            .363           .130          .030
  12.00            .258           .142          .030
  13.00            .154           .123          .030
  14.00            .069           .086          .030
  15.00            .024           .044          .030
  16.00            .027           .014          .030
-99.99
                    └── R(ω_i)_E ──┘            σ_i
WAVE
 &FULLWV
 THETA=22.0,AZIM=90.0,CODIP=12.0,MAGFLD=5.3E-5, IPROFP=0,
 &END
NAME
 &DATUM
 HTSTOP=100.0,50.0,
 STOPS=700.0,3.0,
 NUFLAG=1,
 DOPT=0.6,
 DELTAL=0.0625,
 ROTATE=+90.0,
 MIN=1,
 &END
```

EXAMPLE XXVI

INPUT TO INVERT FOR CASE OF MEASURED DATA

VII. REFERENCES

1. Martin, J. N., and Hildebrand, V. E., Review of Steep-Incidence, VLF, Multiple-Frequency Sounding Techniques and Capabilities. Naval Weapons Center Technical Publication 772, 1968.

2. Budden, K. G., Radio Waves in the Ionosphere, Cambridge University Press, 1961.

3. Morfitt, D. G., and Shellman, C. H., A Technique for Obtaining D-Region Electron Density Profiles from VLF Reflection Coefficients, Defense Nuclear Agency Interim Report No. 781, 16 November 1977, prepared by Naval Ocean Systems Center.

4. Budden, K. F., "The Numberical Solution of Differential Equations Governing Reflection of Long Radio Waves from the Ionosphere," Royal Society of London, Proceedings. Series A: Mathematical and Physical Sciences, V. 227, p. 516-537, 1955.

5. Shellman, C. H., Determination of D-Region Electron Density Distributions from Radio Propagation Data, NELC TR 1856, 23 January 1973.

The following references are also of interest for inversion procedures.

6. Phillips, David L., "A Technique for the Numerical Solution of Certain Integral Equations of the First Kind." J. Ass. Comput. Mach., V. 9, p. 84-97, 1962.

7. Tomey, S., "On the Numerical Solution of Fredholm Integral Equations of the First Kind by Inversion of the Linear System Produced by Quadrature." J. Ass. Comput. Mach., V. 10, p. 97-101, 1963.

8. Lipa, Belinda, "Derivation of Directional Ocean-Wave Spectra by Integral Inversion of Second-Order Radar Echos." Radio Science, V. 12, No. 3, p. 425-434, 1977.

APPENDIX A

FORTRAN LISTING OF THE "FULLWAVE" COMPUTER PROGRAM

```
MORFITT*FULLWAVE(1).MAIN
      1       C       PROGRAM FULLWAVE
      2       C  MAIN
      3       C  FOR MS
      4       C
      5             COMMON/SIG COM/SIGMA(50)
      6             COMMON /R PNCH/NPUNCH
      7             COMMON/PLOT C/IPROFP,IRPLOT,ICALCP
      8             COMMON/RTOLC/RTOL
      9             COMMON/THETA C/THETA
     10             COMMON/FLD COM/AZIM,CODIP,MAG FLD
     11             COMMON/F COM/FLOW,FHI,FINC
     12             COMMON/EXPNU C/COEFNU,EXPNU
     13             COMMON/MPRNT/MPRINT
     14             COMMON/XPRNT/XPRINT
     15             COMMON/EXTRA9/IBUFFR(500)
     16             NAMELIST/DATUM/AZIM,CODIP,MAG FLD,
     17            $           THETA,RTOL,
     18            $           FLOW,FHI,FINC,
     19            $           COEFNU,EXPNU,NUFLAG,
     20            *           IPROFP,IRPLOT,ICALCP,NPUNCH,
     21            $           MPRINT,XPRINT,SIGMA
     22             INTEGER PROF,COLF,WAVE,ADAT,QUIT
     23             REAL MAG FLD
     24             INTEGER XPRINT
     25             DIMENSION IBCD(20)
     26             DATA PROF/'PROF'/,COLF/'COLF'/,IDEN/'ID  '/,
     27            $           WAVE/'WAVE'/,ADAT/' &DA'/,IEND/' &EN'/,
     28            $           QUIT/'QUIT'/
     29             DATA BDATA/'TUM '/,END2/'D   '/,NAME/'NAME'/
     30       C
     31       C
     32             NPUNCH=1
     33             THETA = 0.0
     34             FLOW = 0.5
     35             FHI = 25.0
     36             FINC = 0.5
     37             RTOL = 0.001
     38             MPRINT = 0
     39             XPRINT = 0
     40             IPROFP=1
     41             IRPLOT=1
     42             ICALCP=0
     43       C
     44             COEFNU = 1.816E11
     45             EXPNU = -0.15
     46             NUFLAG = 0
     47       C
     48             PRINT 67
     49          67 FORMAT('1')
     50       C
     51          10 READ 901,IBCD
```

```
52              PRINT 902,IBCD
53              IF (IBCD(1) .EQ. PROF) GO TO 11
54              IF (IBCD(1) .EQ. COLF) GO TO 12
55              IF (IBCD(1) .EQ. IDEN) GO TO 13
56              IF (IBCD(1) .EQ. WAVE) GO TO 40
57              IF (IBCD(1) .EQ. QUIT) GO TO 905
58              IF(IBCD(1) .EQ. NAME) GO TO 90
59              PRINT 900
60            900 FORMAT (' ','ERROR IN CONTROL CARD')
61            905 CALL OVCHK('MAIN  ',905)
62              STOP
63          C
64             11 CALL PRFL IN
65              GO TO 10
66             12 CALL COLF IN
67              GO TO 10
68             13 READ 901,IBCD
69              PRINT 902,IBCD
70              PUNCH 901,IBCD
71              GO TO 10
72          C
73             90 READ(5,DATUM)
74              WRITE(6,550) ADAT,BDATA
75            550 FORMAT(' ',2A4)
76              WRITE(6,555) AZIM,CODIP,MAGFLD,THETA,RTOL
77            555 FORMAT(' ',' AZIM = ',F8.3,' CODIP = ',F8.3,' MAGFLD = ',1PE10.3,
78               * ' THETA = ',0PF10.4,' DEGREES',' RTOL = ',0PF8.4)
79              WRITE(6,444) FLOW,FHI,FINC,NPUNCH
80            444 FORMAT(' ','FLOW = ',F10.4,' KHZ',' FHI = ',F10.4,' KHZ',
81               * '  FINC = ',F10.4,' KHZ',  '  NPUNCH = ',I5)
82              IF(NUFLAG .NE. 1)        WRITE(6,557) NUFLAG,COEFNU,EXPNU
83            557 FORMAT(' ',' NUFLAG = ',I2,' COEFNU = ',1PE11.4,' EXPNU = ',
84               * 0PF9.5)
85              WRITE(6,551) IEND,END2
86            551 FORMAT(' ',2A4,//)
87              IF (NUFLAG .EQ. 0) GO TO 10
88              COEFNU = 4.303E11
89               EXPNU = -0.1622
90              GO TO 10
91          C
92             40 CALL CONTRL
93              GO TO 10
94          C
95            901 FORMAT(20A4)
96            902 FORMAT(' ',20A4)
97          C
98              END
```

```
RT,S FULLWAVE.BDATA
```

```
MORFITT*FULLWAVE(1).BDATA
      1              BLOCK DATA
      2              COMMON/SIG COM/SIGMA(50)
      3              DATA SIGMA/50*0.03/
      4              END

@PRT,S FULLWAVE.PRFLIN
```

```
MORFITT*FULLWAVE(1).PRFLIN
      1              SUBROUTINE PRFL IN
      2        C FOR MS
      3        C
      4              COMMON/PLOT C/IPROFP,IRPLOT,ICALCP
      5              COMMON/NR OF AS/NR A
      6              COMMON/HTS COM/HTS(101)
      7              COMMON/ALOGN C/ALOGEN(101)
      8              COMMON/EXPNU C/COEFNU,EXPNU
      9              DIMENSION IBCD(20),HTS NU(2),VAL NU(2)
     10              DIMENSION Y(101)
     11              DIMENSION LINE(81)
     12              INTEGER V
     13        C
     14              DATA ISYMN/'+'/,IBLANK/' '/
     15              DATA ALN TEN/2.302585/
     16              DATA V/0174000000000/
     17              FMESH=4.0*ALN TEN/80.0
     18        C
     19        C
     20              READ 901,(IBCD(L),L=1,20)
     21              PRINT 902,(IBCD(L),L=1,20)
     22              J = 1
     23          111 READ 904,HT,EN
     24          904 FORMAT (F7.2,5X,E9.2)
     25              IF(HT .LT. 0.0) GO TO 1000
     26              HTS(J) = HT
     27              Y(J) = EN
     28              IF(J .NE. 1 .AND. HTS(J) .GE. HTS(J-1)) STOP
     29              PRINT 906,HT,EN
     30          906 FORMAT (' ',F7.2,5X,1PE9.2)
     31              IF (EN .LT. 1.0E-20) EN = 1.0E-20
     32              ALOGEN(J) = ALOG(EN)
     33              NR A = J
     34              J = J+1
     35              GO TO 111
     36        C
     37        C
     38              ENTRY COLF IN
     39              J = 1
     40           21 READ 905,HT,ENU
     41          905 FORMAT(F7.2,4X,3(1X,1PE9.2))
     42              PRINT 888,HT,ENU
     43          888 FORMAT(' ',F7.2,4X,3(1X,1PE9.2))
     44              IF (HT .LT. 0.0) GO TO 23
     45              IF (J .GE. 3) GO TO 27
     46              HTS NU(J) = HT
     47              VAL NU(J) = ENU
     48              J = J+1
     49              GO TO 21
     50        C
     51           23 EXP NU = ALOG(VAL NU(1)/VAL NU(2))/(HTS NU(1)-HTS NU(2))
```

```
      52              COEFNU = VAL NU(2)*EXP(-EXP NU*HTS NU(2))
      53              RETURN
      54         1000 CONTINUE
      55       C
      56       C      IF(ICALCP .EQ. 0) GO TO 75
      57       C      CALL BGNPL(1)
      58       C      CALL TITLE(IBCD,40,'ELECTRONS',9,'ALTITUDE',8,5.,5.)
      59       C      CALL XLOG(.1,1.,0.,20.)
      60       C      CALL CURVE(Y,HTS,NRA,0)
      61       C      CALL ENDPL(1)
      62       C
      63          75  IF(IPROFP .EQ. 0) GO TO 76
      64              PRINT 60
      65          60  FORMAT('1')
      66              PRINT 100,V,V,V,V,V
      67         100  FORMAT ('0',5X,'HT',9X,'N',18X, A1,19('-'), A1,19('-'),
      68             *  A1,19('-'), A1,19('-'), A1)
      69              DO 11 J=1,NP A
      70              DO 50 K = 1,81
      71          50  LINE(K) = IBLANK
      72              JPOST=ALOGEN(J)/FMESH+1.5
      73              IF(JPOST .GE. 1 .AND. JPOST .LE. 81)
      74             *LINE(JPOST)=ISYMN
      75          11  PRINT 105,HTS(J),Y(J),V,LINE,V
      76         105  FORMAT (' ',F7.2,1X,1PE9.2,17X, A1,81A1, A1)
      77              PRINT 501,V,V,V,V,V
      78         501  FORMAT(' ',35X, A1,19('-'), A1,19('-'), A1,19('-'), A1,
      79             * 19('-'), A1 )
      80              PRINT 502
      81         502  FORMAT(' ',35X,'0',19X,'1',19X,'2',19X,'3',19X,'4')
      82       C
      83          76  CONTINUE
      84              RETURN
      85       C
      86          27  PRINT 907
      87         907  FORMAT ('0','TWO HTS ONLY ALLOWED IN NU PROFILE')
      88              STOP
      89       C
      90         901  FORMAT (20A4)
      91         902  FORMAT (' ',20A4)
      92       C
      93              END

@PRT,S FULLWAVE.INITLR
```

```
MORFITT*FULLWAVE(1).INITLR
      1         SUBROUTINE INITL R
      2   C FOR MS
      3   C
      4         COMMON/XPRNT/XPRINT
      5         COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
      6         COMMON/INTEGR/R11,R21,R12,R22
      7         COMMON/C COM/C,S
      8         COMPLEX
      9        $           M11,M21,M31,M12,M22,M32,M13,M23,M33,
     10        $           R11,R21,R12,R22,
     11        $           QTEMP(4),EIPI04,Q(2),
     12        $           B4,B3,B2,B1,B0,
     13        $           D11,D12,D13,D31,D32,D33,
     14        $           P(2),T(2),
     15        $           DEN,FACTOR
     16         INTEGER XPRINT
     17         DATA EIPI04/(0.707107,0.707107)/
     18   C
     19   C
     20         CSQ = C**2
     21   C
     22         B4 = 1.0+M33
     23         B3 = S*(M13+M31)
     24         B2 = -(CSQ+M33)*(1.0+M11)+M13*M31-(1.0+M33)*(CSQ+M22)+M23*M32
     25         B1 = S*(M12*M23+M21*M32-(CSQ+M22)*(M13+M31))
     26         B0 = (1.0+M11)*(CSQ+M22)*(CSQ+M33)+M12*M23*M31+M13*M21*M32
     27        $           -M13*(CSQ+M22)*M31-(1.0+M11)*M23*M32-M12*M21*(CSQ+M33)
     28   C
     29         CALL QUARTC(B4,B3,B2,B1,B0,QTEMP)
     30   C
     31         K = 0
     32         DO 21 KT=1,4
     33         IF (REAL(EIPI04*QTEMP(KT)) .LT. 0.0) GO TO 21
     34         K = K+1
     35         IF (K .GE. 3) GO TO 90
     36         Q(K) = QTEMP(KT)
     37      21 CONTINUE
     38         IF (K .NE. 2) GO TO 90
     39   C
     40         IF(XPRINT .NE. 0) PRINT 901,(Q(J),J=1,2)
     41    901  FORMAT('0','Q= ',2(E15.4,E13.4),/)
     42   C
     43         DO 31 J=1,2
     44         D11 = 1.0+M11-Q(J)**2
     45         D12 = M12
     46         D13 = M13+Q(J)*S
     47         D31 = M31+Q(J)*S
     48         D32 = M32
     49         D33 = CSQ+M33
     50         DEN = D11*D33-D13*D31
     51         P(J) = (-D12*D33+D13*D32)/DEN
```

```
      31 T(J) = Q(J)*P(J)-S*(-D11*D32+D12*D31)/DEN
C
      DEN = (T(1)*C+P(1))*(C+Q(2))-(T(2)*C+P(2))*(C+Q(1))
      FACTOR = 2.0/DEN
      R11 = (T(1)*(C+Q(2))-T(2)*(C+Q(1)))*FACTOR
      R22 = ((T(1)*C+P(1))-(T(2)*C+P(2)))*FACTOR
      R12 = -(T(1)*P(2)-T(2)*P(1))*FACTOR
      R21 = -(Q(1)-Q(2))*FACTOR
      RETURN
C
      90 PRINT 900
     900 FORMAT('0','PROBLEM IN SORTING Q VALUES')
      STOP
C
      END

&PRT,S FULLWAVE.TMTRX
```

```
MORFITT*FULLWAVE(1).TMTRX
      1         SUBROUTINE T MTRX
      2   C FOR MS
      3   C
      4         COMMON/FRQ COM/FREQ
      5         COMMON/FLD COM/AZIM,CODIP,MAG FLD
      6         COMMON/WN/WAVE NR
      7         COMMON/EN COLL/HT,COLL,EN
      8         COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
      9         COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
     10        $           X31(001),X42(001),X32(001),X41(001)
     11         COMPLEX
     12        $           U,USQ,D,IUD,
     13        $           USQD,TA,TB,
     14        $           D TEMP,
     15        $           M13D,M31D,M23D,M32D,
     16        $           M2332D,M1331D,M2331D,M3213D,
     17        $           M11,M21,M31,M12,M22,M32,M13,M23,M33,
     18        $           X11,X44,X12,X34,X14,X31,X42,X32,X41
     19         REAL MAG FLD,LY,MY,NY,IUD PRT,
     20        $           LSQYSQ,MSQYSQ,NSQYSQ,LMYSQ,LNYSQ,MNYSQ
     21         DIMENSION U PARTS(2),USQ PRT(2),D PARTS(2),IUD PRT(2)
     22         EQUIVALENCE (U,U PARTS(1)),(USQ,USQ PRT(1)),
     23        * (D,D PARTS(1)),(IUD,IUD PRT(1))
     24         DATA PI/3.141592653/
     25         DATA DTR/0.01745329252/
     26         DATA COEFF X/3.182357E03/,COEFF Y/1.758796E11/
     27         DATA VEL LT/2.997928E05/
     28   C
     29   C
     30         CALL EN NU
     31   C
     32         X = COEF EN*EN
     33         Z = COLL*OV OMGA
     34         U PARTS(1) = 1.0
     35         U PARTS(2) = -Z
     36         USQ PRT(1) = 1.0-Z**2
     37         USQ PRT(2) = -Z-Z
     38         D = -X/(U*(USQ-YSQ))
     39         IUD PRT(1) = Z*D PARTS(1)-D PARTS(2)
     40         IUD PRT(2) = D PARTS(1)+Z*D PARTS(2)
     41         USQD = USQ*D
     42   C
     43         M11 = USQD-LSQYSQ*D
     44         M22 = USQD-MSQYSQ*D
     45         M33 = USQD-NSQYSQ*D
     46         TA = NY*IUD
     47         TB = LMYSQ*D
     48         M21 = +TA-TB
     49         M12 = -TA-TB
     50         TA = MY*IUD
     51         TB = LNYSQ*D
```

```
      M13 = +TA-TB
      M31 = -TA-TB
      TA = LY*IUD
      TB = MNYSQ*D
      M32 = +TA-TB
      M23 = -TA-TB
C
      D = 1.0/(1.0+M33)
      M13D = M13*D
      M31D = M31*D
      M23D = M23*D
      M32D = M32*D
      M2332D = M32*M23D
      M1331D = M31*M13D
      M2331D = M31*M23D
      M3213D = M32*M13D
C
      L = 1
      X11(L) = -M31D
      X44(L) = -M13D
      X12(L) = M32D
      X34(L) = M23D
      X14(L) = D
      X31(L) = M2331D-M21
      X42(L) = M3213D-M12
      X32(L) = M22-M2332D
      X41(L) = 1.0+M11-M1331D
      RETURN
C
C
      ENTRY INIT T
      OMEGA = 2.0*PI*FREQ*1000.0
      WAVE NR = OMEGA/VEL LT
      COEF EN = COEFF X*1.0E06/OMEGA**2
      OV OMGA = 1.0/OMEGA
      SIN DIP = SIN(CODIP*DTR)
      DIR CS L = SIN DIP*COS(AZIM*DTR)
      DIR CS M = SIN DIP*SIN(AZIM*DTR)
      DIR CS N = -COS(CO DIP*DTR)
      Y = -COEFF Y*MAG FLD/OMEGA
      YSQ = Y**2
      LY = DIR CS L*Y
      MY = DIR CS M*Y
      NY = DIR CS N*Y
      LSQYSQ = DIR CS L**2*YSQ
      MSQYSQ = DIR CS M**2*YSQ
      NSQYSQ = DIR CS N**2*YSQ
      LMYSQ = DIR CS L*DIR CS M*YSQ
      LNYSQ = DIR CS L*DIR CS N*YSQ
      MNYSQ = DIR CS M*DIR CS N*YSQ
      RETURN
C
```

```
104          END

@PRT,S FULLWAVE.ENNU
```

```
MORFITT*FULLWAVE(1).ENNU
      1         SUBROUTINE EN NU
      2   C FOR MS
      3   C
      4         COMMON/NR OF AS/NR A
      5         COMMON/JAY COM/J
      6         COMMON/HTS COM/HTS(101)
      7         COMMON/ALOGN C/ALOGEN(101)
      8         COMMON/EXPNU C/COEFNU,EXPNU
      9         COMMON/EN COLL/HT,ENU,EN
     10   C
     11   C
     12         F1 = (HT-HTS(J+1))*FACTR
     13         F2 = (HTS(J)-HT)*FACTR
     14         EN = EXP(ALOGEN(J)*F1+ALOGEN(J+1)*F2)
     15         ENU = COEFNU*EXP(EXPNU*HT)
     16         RETURN
     17   C
     18   C
     19         ENTRY SET FTR
     20         FACTR = 1.0/(HTS(J)-HTS(J+1))
     21         RETURN
     22   C
     23         END

@PRT,S FULLWAVE.INTEG
```

```
MORFITT*FULLWAVE(1).INTEG
      1         SUBROUTINE INTEG
      2   C FOR MS
      3   C
      4         COMMON/RTOLC/RTOL
      5         COMMON/NR OF AS/NR A
      6         COMMON/HTS COM/HTS(101)
      7         COMMON/XPRNT/XPRINT
      8         COMMON/INTEGR/R(8),DRDH(8)
      9         COMMON/X INTGR/X(8),DXDH(8)
     10         COMMON/EN COLL/HT
     11         COMMON/WN/WAVE NR
     12         COMMON/OVRFLO/IOVFLO
     13         COMMON/JAY COM/JAY
     14         DIMENSION
     15        $           R0(16),HDELR0(16),DELR1(16),DELR2(16),
     16        $           X0(8),HDELX0(8),DELX1(8),DELX2(8),
     17        $           R SAVE(16),
     18        $           RKHTS(401)
     19         INTEGER XPRINT
     20         DATA MAX NR/401/
     21         DATA DHMIN/0.01/
     22   C
     23   C
     24         N = 8
     25         IOVFLO = 0
     26         IF(XPRINT .NE. 0) CALL R COLS
     27         THIRD = 1.0/3.0
     28         CALL INIT T
     29         JAY = 1
     30         JRK = 1
     31         HT = HTS(1)
     32         CALL SET FTR
     33         CALL S MTRX
     34         CALL INITL R
     35   C
     36         CALL XFER (R,X,8)
     37         IF(XPRINT .NE. 0) PRINT 901
     38     901 FORMAT ('0')
     39         IF(XPRINT .NE. 0) CALL PRINT R
     40   C
     41         NRAM1 = NR A-1
     42         DO 79 J=1,NRAM1
     43         JAY = J
     44         CALL SET FTR
     45      30 CALL XFER(R,R SAVE,N)
     46         HT = RKHTS(JRK)
     47         CALL XFER (R,X,8)
     48      31 DELH = (RKHTS(JRK+1)-RKHTS(JRK))*0.5
     49         IF (ABS(DELH) .LT. DHMIN) GO TO 90
     50         DH = DELH*WAVE NR/2.0
     51         HDH = 0.5*DH
```

```
      TDH = 2.0*DH
C
      CALL R DERIV
      IF (IOVFLO .NE. 0) GO TO 60
C
      DO 32 I=1,N
      RO(I) = R(I)
      HDELRO(I) = DRDH(I)*HDH
   32 R(I) = RO(I)+HDELRO(I)
C
      DO 33 I=1,8
      XO(I) = X(I)
      HDELXO(I) = HDELRO(I)*2.0
   33 X(I) = XO(I)+HDELXO(I)
C
      HT = HT+0.5*DELH
      CALL S MTRX
      CALL R DERIV
      IF(IOVFLO .NE. 0) GO TO 60
C
      DO 34 I=1,N
      DELR1(I) = DRDH(I)*DH
   34 R(I) = RO(I)+0.5*DELR1(I)
C
      CALL R DERIV
      IF(IOVFLO .NE. 0) GO TO 60
C
      DO 35 I=1,N
      DELR2(I) = DRDH(I)*DH
   35 R(I) = RO(I)+DELR2(I)
C
      HT = HT+0.5*DELH
      CALL S MTRX
      CALL R DERIV
      CALL X DERIV
      IF(IOVFLO .NE. 0) GO TO 60
C
      DO 36 I=1,N
      HDELR3 = DRDH(I)*HDH
      DELR4 = (HDELRO(I)+DELR1(I)+DELR2(I)+HDELR3)*THIRD
   36 R(I) = RO(I)+DELR4
C
      DO 37 I=1,8
      DELX1(I) = DXDH(I)*TDH
   37 X(I) = XO(I)+0.5*DELX1(I)
C
      CALL R DERIV
      CALL X DERIV
      IF(IOVFLO .NE. 0) GO TO 60
C
      DO 42 I=1,N
      RO(I) = R(I)
```

```
104         HDELR0(I) = DRDH(I)*HDH
105      42 R(I) = R0(I)+HDELR0(I)
106   C
107         DO 43 I=1,8
108         DELX2(I) = DXDH(I)*TDH
109      43 X(I) = X0(I)+DELX2(I)
110   C
111         HT = HT+0.5*DELH
112         CALL S MTRX
113         CALL R DERIV
114         IF(IOVFLO .NE. 0) GO TO 60
115   C
116         DO 44 I=1,N
117         DELR1(I) = DRDH(I)*DH
118      44 R(I) = R0(I)+0.5*DELR1(I)
119   C
120         CALL R DERIV
121         IF(IOVFLO .NE. 0) GO TO 60
122   C
123         DO 45 I=1,N
124         DELR2(I) = DRDH(I)*DH
125      45 R(I) = R0(I)+DELR2(I)
126   C
127         HT = HT+0.5*DELH
128         CALL S MTRX
129         CALL R DERIV
130         CALL X DERIV
131         IF(IOVFLO .NE. 0) GO TO 60
132   C
133         DO 46 I=1,N
134         HDELR3 = DRDH(I)*HDH
135         DELR4 = (HDELR0(I)+DELR1(I)+DELR2(I)+HDELR3)*THIRD
136      46 R(I) = R0(I)+DELR4
137   C
138         DO 47 I=1,8
139         HDELX3 = DXDH(I)*DH
140         DELX4 = (HDELX0(I)+DELX1(I)+DELX2(I)+HDELX3)*THIRD
141      47 X(I) = X0(I)+DELX4
142   C
143         IF(XPRINT .NE. 0) CALL PRINT R
144         DO 51 I=1,7,2
145         ERRSQ = (R(I)-X(I))**2+(R(I+1)-X(I+1))**2
146         RSQ = R(I)**2+R(I+1)**2
147         IF (RSQ .GT. 1.0) ERRSQ = ERRSQ/RSQ
148         IF (ERRSQ .GT. RTOL**2) GO TO 61
149      51 CONTINUE
150         GO TO 70
151   C
152      60 IF (XPRINT .NE. 0) CALL PRINT R
153      61 IF (XPRINT .NE. 0) PRINT 960
154     960 FORMAT (' ','OVRFLO')
155         IF (NR RK .GE. MAX NR) GO TO 95
```

```
      156          IOVFLO = 0
      157          CALL XFER(R SAVE,R,N)
      158          CALL XFER (R SAVE,X,8)
      159          HT = RKHTS(JRK)
      160          CALL S MTRX
      161    C
      162          NRRKM1 = NR RK-1
      163          DO 62 JCOUNT=JRK,NRRKM1
      164          JJ = NRRKM1-JCOUNT+JRK
      165       62 RKHTS(JJ+2) = RKHTS(JJ+1)
      166          RKHTS(JRK+1) = (RKHTS(JRK)+RKHTS(JRK+2))*0.5
      167          NR RK = NR RK+1
      168          GO TO 31
      169    C
      170       70 JRK = JRK+1
      171          IF (ABS(RKHTS(JRK)-HTS(J+1)) .GT. 0.001) GO TO 30
      172       79 CONTINUE
      173          RETURN
      174    C
      175    C
      176          ENTRY SET RK
      177          DO 81 J=1,NR A
      178       81 RKHTS(J) = HTS(J)
      179          NR RK = NR A
      180          RETURN
      181    C
      182       90 PRINT 990
      183      990 FORMAT ('0','STEP TOO SMALL IN INTEG')
      184          STOP
      185    C
      186       95 PRINT 995
      187      995 FORMAT ('0','TOO MANY STEPS IN INTEG')
      188          STOP
      189    C
      190          END

@PRT,S FULLWAVE.ROUT
```

```
MORFITT*FULLWAVE(1).ROUT
      1         SUBROUTINE R OUT
      2   C FOR MS
      3   C
      4         COMMON/INTEGR/R(8)
      5         COMMON/X INTGR/X(8)
      6         COMMON/EN COLL/HT,EC OMIT(2)
      7   C
      8   C
      9         ENTRY R COLS
     10         PRINT 900
     11     900 FORMAT('0',8X,'HT',10X,'11R11',16X,'11R1',16X,'1R11',17X,'1R1')
     12         RETURN
     13   C
     14   C
     15         ENTRY PRINT R
     16         PRINT 901,HT,(X(I),I=1,8)
     17         PRINT 901,HT,(R(I),I=1,8)
     18     901 FORMAT(' ',F10.2,4(2X,F9.5,F9.5))
     19         PRINT 902
     20     902 FORMAT ('0')
     21         RETURN
     22   C
     23         END

@PRT,S FULLWAVE.SMTRX
```

```
MORFITT*FULLWAVE(1).SMTRX
      1         SUBROUTINE S MTRX
      2   C   FOR MS
      3   C
      4         COMMON/THETA C/THETA
      5         COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
      6        $           X31(001),X42(001),X32(001),X41(001)
      7         COMMON/C COM/C,S
      8         COMMON/S MTX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22
      9         COMPLEX
     10        $           X11,X44,X12,X34,X14,X31,X42,X32,X41,
     11        $           TEMP,TWOC,
     12        $           A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22
     13         DATA RTD/57.29578/
     14   C
     15   C
     16         CALL T MTRX
     17   C
     18         L = 1
     19         TEMP = X41(L)+X41(L)
     20         A11 = TEMP+TEMP
     21         A22 = 4.0
     22         TEMP = S*X44(L)-C*X41(L)
     23         B11 = TEMP+TEMP
     24         B12 = -X42(L)-X42(L)
     25         TWOC = C+C
     26         B22 = -TWOC
     27         TEMP = -S*X11(L)-C*X41(L)
     28         C11 = TEMP+TEMP
     29         C21 = X31(L)+X31(L)
     30         C22 = -TWOC
     31         D11 = CS*(X11(L)-X44(L))-(1.0-X14(L))+CSQ*(X41(L)-X14(L))
     32         D21 = -C*X31(L)+S*X34(L)
     33         D12 = S*X12(L)+C*X42(L)
     34         D22 = -X32(L)
     35         RETURN
     36   C
     37   C
     38         ENTRY INIT S
     39         C = COS(THETA/RTD)
     40         CSQ = C**2
     41         S = SIN(THETA/RTD)
     42         CS = C*S
     43         RETURN
     44   C
     45         END

@PRT,S FULLWAVE.DIFFEQ
```

```
MORFITT*FULLWAVE(1).DIFFEQ
      1         SUBROUTINE DIFF EQ
      2   C FOR MS
      3   C
      4         COMMON/INTEGR/R(8),DRDH(8)
      5         COMMON/X INTGR/X(8),DXDH(8)
      6         COMMON/S MTX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22
      7         COMMON/EXTRA7/R11,R21,R12,R22,
      8        $            DR11DH,DR21DH,DR12DH,DR22DH
      9         COMMON/OVRFLO/IOVFLO
     10         COMPLEX
     11        $            R11,R21,R12,R22,
     12        $            DR11DH,DR21DH,DR12DH,DR22DH,
     13        $            A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
     14        $            D11R11,D12R21,D21R12,D22R22,
     15        $            R11R22,R12R21
     16         DIMENSION R MTRX(16),DERIV(16)
     17         EQUIVALENCE(R11,R MTRX),(DR11DH,DERIV)
     18   C
     19   C
     20         ENTRY R DERIV
     21         CALL XFER (R,R MTRX,8)
     22         IXFLAG = 0
     23      20 DO 21 I=1,8
     24         IF(ABS(R MTRX(I)) .GT. 1.0E4) GO TO 90
     25      21 CONTINUE
     26   C
     27         D11R11 = D11*R11
     28         D12R21 = D12*R21
     29         D21R12 = D21*R12
     30         D22R22 = D22*R22
     31         R11R22 = R11*R22
     32         R12R21 = R12*R21
     33   C
     34         DR11DH = A11+(B11+C11+D11R11+D12R21+D21R12)*R11
     35        $           +B12*R21+C21*R12+D22*R12R21
     36         DR21DH = (B22+C11+D11R11+D12R21+D22R22)*R21
     37        $           +C21*R22+D21*R11R22
     38         DR12DH = (B11+C22+D11R11+D21R12+D22R22)*R12
     39        $           +B12*R22+D12*R11R22
     40         DR22DH = A22+(B22+C22+D12R21+D21R12+D22R22)*R22
     41        $           +D11*R12R21
     42         IF (IXFLAG .NE. 0) GO TO 40
     43   C
     44         DO 24 I=1,7,2
     45         DRDH(I) = DERIV(I+1)
     46      24 DRDH(I+1) = -DERIV(I)
     47         RETURN
     48   C
     49   C
     50         ENTRY X DERIV
     51         CALL XFER (X,R MTRX,8)
```

```
      IXFLAG = 1
      GO TO 20
C
   40 DO 42 I=1,7,2
      DXDH(I) = DERIV(I+1)
   42 DXDH(I+1) = -DERIV(I)
      RETURN
C
   90 IOVFLO = 1
      RETURN
C
      END
```

```
@PRT,S FULLWAVE.CONTRL
```

```
MORFITT*FULLWAVE(1).CONTRL
      1                SUBROUTINE CONTRL
      2        C
      3                COMMON/SIG COM/SIGMA(50)
      4                COMMON /R PNCH/NPUNCH
      5                COMMON/NRF COM/NPOINT
      6                COMMON/PLOT C/IPROFP,IRPLOT,ICALCP
      7                COMMON/RDATA C/X(50),Y(50),F(50)
      8                COMMON/F COM/FLOW,FHI,FINC
      9                COMMON/FRQ COM/FREQ
     10                COMMON/INTEGR/R11,R21,R12,R22
     11                COMMON/C COM/C
     12                COMPLEX R11,R21,R12,R22,
     13               $           CPLX I,R
     14                DATA CPLX I/(0.0,1.0)/
     15                ALAST=-99.9
     16        C
     17        C
     18                PRINT 900
     19          900   FORMAT ('1',6X,'F',13X,'R',16X,'SIGMA',/)
     20                CALL INIT S
     21                CALL SET RK
     22        C
     23                FREQ = FLOW
     24                NPOINT = 0
     25             11 CALL INTEG
     26                NPOINT = NPOINT+1
     27                F(NPOINT)=FREQ
     28                R11 = C*R11-1.0
     29                R21 = C*R21
     30                R12 = C*R12
     31                R22 = C*R22-1.0
     32                R = (-R22+CPLX I*R12)/(-R22-CPLX I*R12)
     33                X(NPOINT) = REAL(R)
     34                Y(NPOINT) = AIMAG(R)
     35                PRINT 910,FREQ,R,SIGMA(NPOINT)
     36          910   FORMAT (' ',F7.2,3X,2F9.3,3X,F10.3)
     37                IF(NPUNCH .GT. 0) PUNCH 103,FREQ,R,SIGMA(NPOINT)
     38          103   FORMAT(F7.2,3X,2F10.3,F10.3)
     39        C
     40                FREQ = FREQ+FINC
     41                IF(FREQ .LE. FHI) GO TO 11
     42                PRINT 910,ALAST
     43                IF(NPUNCH .GT. 0) PUNCH 103,ALAST
     44        C
     45        C       IF(ICALCP .EQ. 0) GO TO 75
     46        C       CALL BGNPL(1)
     47        C       CALL CROSS
     48        C       CALL TITLE(' ',1,'REAL',4,'IMAG',4,8.,8.)
     49        C       CALL GRAPH(-1.,.25,-1.,.25)
     50        C       CALL MARKER(3)
     51        C       CALL CURVE(X,Y,NPOINT,1)
```

```
      C     CALL ENDPL(1)
      C
         75 IF(IRPLOT .EQ. 0) GO TO 76
            PRINT 60
         60 FORMAT('1')
            CALL RPLOTS
      C
         76 CONTINUE
            RETURN
      C
            END
```

```
@PRT,S FULLWAVE.QUARTC
```

```
MORFITT*FULLWAVE(1).QUARTC
      1         SUBROUTINE QUARTC(B4,FOUR B3,SIX B2,FOUR B1,ONE B0,Q)
      2   C FOR MS
      3   C
      4         COMPLEX FOUR B3,SIX B2,FOUR B1,ONE B0,Q,B4,
      5        $           H,I,G,H PRIME,G PRIME,
      6        $           SQ ROOT,P POS,P,LOG P,
      7        $           CUB RT0,CUB RT1,CUB RT2,OMEGA1,OMEGA2,
      8        $           ROOT P,ROOT Q,ROOT R,
      9        $           TWLV B3,TWLV B2,F,DFDQ,DEL Q,FB4
     10         COMPLEX*16 B3,B2,B1,B0,B3 SQ
     11         REAL MAG POS,MAG NEG
     12         DIMENSION Q(4),P RI(2)
     13         EQUIVALENCE(P,P RI)
     14         DATA OMEGA1/(-0.5,0.8660254038)/,OMEGA2/(-0.5,-0.8660254038)/
     15         DATA PRECSN/1.0E-4/
     16   C
     17   C
     18         B3 = FOUR B3/(4.0*B4)
     19         B2 = SIX B2/(6.0*B4)
     20         B1 = FOUR B1/(4.0*B4)
     21         B0 = ONE B0/B4
     22   C
     23         B3 SQ = B3**2
     24         H = B2-B3 SQ
     25         I = B0-4.0*B3*B1+3.0*B2**2
     26         G = B1+B3*(-3.0*B2+2.0*B3 SQ)
     27         H PRIME = -I/12.0
     28         G PRIME = -G**2/4.0-H*(H**2+3.0*H PRIME)
     29   C
     30         SQ ROOT = CSQRT(G PRIME**2+4.0*H PRIME**3)
     31         P = (-G PRIME+SQ ROOT)*0.5
     32         MAG POS = ABS(P RI(1))+ABS(P RI(2))
     33         P POS = P
     34         P = (-G PRIME-SQ ROOT)*0.5
     35         MAG NEG = ABS(P RI(1))+ABS(P RI(2))
     36         IF(MAG POS .GT. MAG NEG) P = P POS
     37         LOG P = CLOG(P)
     38         CUB RT0 = CEXP(LOG P/3.0)
     39         CUB RT1 = OMEGA1*CUB RT0
     40         CUB RT2 = OMEGA2*CUB RT0
     41   C
     42         ROOT P = CSQRT(CUB RT0-H PRIME/CUB RT0-H)
     43         ROOT Q = CSQRT(CUB RT1-H PRIME/CUB RT1-H)
     44         ROOT R = CSQRT(CUB RT2-H PRIME/CUB RT2-H)
     45   C     IF(CABS(G) .LT. 1.0E-50) GO TO 21                          TJM
     46         IF(CABS(G) .LT. 1.0E-36) GO TO 21                          TJM
     47         SIGN = -ROOT P*ROOT Q*ROOT R*2.0/G
     48         IF(SIGN .LT. 0.0) ROOT R = -ROOT R
     49        21 Q(1) = +ROOT P+ROOT Q+ROOT R-B3
     50           Q(2) = +ROOT P-ROOT Q-ROOT R-B3
     51           Q(3) = -ROOT P+ROOT Q-ROOT R-B3
```

```
      52          Q(4) = -ROOT P-ROOT Q+ROOT R-B3
      53    C
      54          FB4 = 4.0*B4
      55          TWLV B3 = 3.0*FOUR B3
      56          TWLV B2 = 2.0*SIX B2
      57          DO 52 J=1,4
      58          ITER = 0
      59       51 F = (((B4*Q(J)+FOUR B3)*Q(J)+SIX B2)*Q(J)+FOUR B1)*Q(J)+ONE B0
      60          DFDQ = ((FB4*Q(J)+TWLV B3)*Q(J)+TWLV B2)*Q(J)+FOUR B1
      61          DEL Q = -F/DFDQ
      62          Q(J) = Q(J)+DEL Q
      63          IF(CABS(DEL Q/Q(J)) .LT. PRECSN) GO TO 52
      64          ITER = ITER+1
      65          IF(ITER .GE. 10) GO TO 90
      66          GO TO 51
      67       52 CONTINUE
      68          RETURN
      69    C
      70       90 PRINT 900
      71      900 FORMAT('0','Q FAILS TO CONVERGE IN QUARTC')
      72          STOP
      73    C
      74          END

@PRT,S FULLWAVE.XFER
```

```
MORFITT*FULLWAVE(1).XFER
      1         SUBROUTINE XFER (A,B,N)
      2   C FOR MS
      3   C
      4         DIMENSION A(1),B(1)
      5   C
      6   C
      7         DO 11 J=1,N
      8      11 B(J) = A(J)
      9         RETURN
     10   C
     11         END

@PRT,S FULLWAVE.RPLOTS
```

```
MORFITT*FULLWAVE(1).RPLOTS
      1         SUBROUTINE RPLOTS
      2   C
      3         COMMON/RDATA C/X(50),Y(50),F(50)
      4         COMMON/NRF COM/NR F
      5         DIMENSION ISYMD(50),XD(50),YD(50),
      6        * LINE(101)
      7         INTEGER V
      8   C
      9         DATA ISYMD/'1','2','3','4','5','6','7','8','9','0',
     10        *           'A','B','C','D','E','F','G','H','I','J',
     11        *           'K','L','M','N','O','P','Q','R','S','T',
     12        *           'U','V','W','X','Y','Z','1','2','3','4'/
     13         DATA IBLANK/' '/
     14         DATA V/01740000000000/
     15   C
     16         XMAX=-1.0E9
     17         XMIN=+1.0E9
     18         YMAX=-1.0E9
     19         YMIN=+1.0E9
     20   C
     21         DO 10 L=1,NR F
     22         XD(L)=X(L)
     23         YD(L)=Y(L)
     24         IF(XD(L) .LT. XMIN) XMIN=XD(L)
     25         IF(XD(L) .GT. XMAX) XMAX=XD(L)
     26         IF(YD(L) .LT. YMIN) YMIN=YD(L)
     27         IF(YD(L) .GT. YMAX) YMAX=YD(L)
     28      10 CONTINUE
     29   C
     30         IF((XMAX-XMIN)/10.0 .GT. (YMAX-YMIN)/6.5) GO TO 100
     31         YMESH=(YMAX-YMIN)/52.0
     32         XMESH=YMESH*0.8
     33         XMAX=XMIN+(100.0*XMESH)
     34         GO TO 200
     35     100 XMESH=(XMAX-XMIN)/100.0
     36         YMESH=XMESH/0.8
     37         YMAX=YMIN+(52.0*YMESH)
     38     200 CONTINUE
     39         PRINT 30
     40      30 FORMAT('1',' PLOT OF R-VALUES')
     41         YTOP=YMAX+YMESH
     42         PRINT 31,YTOP
     43      31 FORMAT(' ',F10.4,101('-') )
     44         DO 300 I=1,53
     45         DO 350 J=1,101
     46     350 LINE(J) = IBLANK
     47         DO 400 L=1,NR F
     48         ITEST = (YD(L)-YMIN)/YMESH+0.5
     49         IF((ITEST +I) .NE. 53) GO TO 400
     50         JPOST = (XD(L)-XMIN)/XMESH+1.5
     51         LINE(JPOST) = ISYMD(L)
```

```
 52      400 CONTINUE
 53  C
 54      300 PRINT 555,V,LINE,V
 55      555 FORMAT(' ',9X, A1,101A1, A1)
 56          YBOT=YMIN-YMESH
 57          PRINT 32,YBOT
 58       32 FORMAT(' ',F10.4,101('-') )
 59          PRINT 33,XMIN,XMAX
 60       33 FORMAT(' ',5X,F10.4,90X,F10.4)
 61  C
 62          RETURN
 63          END

@PRT,S FULLWAVE.OVCHK
```

```
MORFITT*FULLWAVE(1).OVCHK
      1         SUBROUTINE OVCHK(CALLER,STMT)                                  DMP
      2   C ***** THIS SUBROUTINE WILL CALL THE ROUTINE OVUNFL WHICH           DMP
      3   C ***** CHECKS FOR OVERFLOW AND UNDERFLOW.  THE ROUTINE RETURNS AN   DMP
      4   C ***** INTEGER VALUE WHICH TELLS WHETHER OVERFLOW AND/OR UNDERFLOW  DMP
      5   C ***** HAVE OCCURRED SINCE THE PROGRAM STARTED OR SINCE THE LAST TIME  DMP
      6   C ***** ONE OF THE OVERFLOW/UNDERFLOW CHECKING ROUTINES WAS CALLED,  DMP
      7   C ***** WHICHEVER HAS BEEN MORE RECENT. THE INTEGER VALUES RETURNED  DMP
      8   C ***** AND THEIR MEANINGS ARE AS FOLLOWS:                          DMP
      9   C *****      1= OVERFLOW ONLY                                       DMP
     10   C *****      2= NO OVERFLOW OR UNDERFLOW                            DMP
     11   C *****      3= UNDERFLOW ONLY                                      DMP
     12   C *****      4= OVERFLOW AND UNDERFLOW                              DMP
     13   C ***** THIS SUBROUTINE (OVCHK) WILL PRINT OUT A MESSAGE STATING    DMP
     14   C ***** WHETHER OVERFLOW HAS OR HAS NOT OCCURRED. FOR PURPOSES      DMP
     15   C ***** OF THIS CONVERSION, UNDERFLOW OCCURRENCES HAVE BEEN IGNORED  DMP
     16   C ***** BECAUSE THE 1110 TAKES THE SAME ACTION (I.E. SETS THE RESULT  DMP
     17   C ***** TO ZERO) IN THE CASE OF UNDERFLOW AS DOES THE 360.          DMP
     18   C ***** THE SUBROUTINE ALSO PRINTS OUT THE NAME OF THE CALLING      DMP
     19   C ***** ROUTINE (VARIABLE 'CALLER') AND THE STATEMENT NUMBER FROM    DMP
     20   C ***** WHICH IT WAS CALLED (VARIABLE 'STMT')                       DMP
     21         CHARACTER*6 CALLER                                             DMP
     22         INTEGER STMT                                                   DMP
     23         CALL OVUNFL(IOVFL)                                             DMP
     24         IF (IOVFL .EQ. 1 .OR. IOVFL .EQ. 4) GO TO 150                  DMP
     25         WRITE (6,100)                                                  DMP
     26    100  FORMAT ('1OVERFLOW HAS NOT OCCURRED')                         DMP
     27         GO TO 250                                                      DMP
     28    150  WRITE (6,200)                                                  DMP
     29    200  FORMAT ('1OVERFLOW HAS OCCURRED')                             DMP
     30    250  CONTINUE                                                       DMP
     31         WRITE (6,300) STMT,CALLER                                      DMP
     32    300  FORMAT (' OVCHK WAS CALLED FROM STATEMENT ',I5,' IN SUBROUTINE '  DMP
     33        1   ,A6)                                                       DMP
     34         RETURN                                                         DMP
     35         END                                                            DMP

@FIN
```

APPENDIX B

FORTRAN LISTING OF THE "RANDOM" COMPUTER PROGRAM

PROGRAM RANDOM

```
C        PROGRAM ADD ERROR
C
C      THIS PROGRAM GENERATES GAUSSIAN RANDOM DATA VALUES
C
      COMPLEX PARTS,R,RR
      DIMENSION R(20),RR(20),FREQ(20),SIGMA(20),PART(2),
     $ RDMR(20),RDMI(20),IRDMR(20),IRDMI(20),IDENT(20)
      EQUIVALENCE (PART(1),PARTS)
      NAMELIST/DATUM/ IRANDM,NRF,SIGMA
C
C        DEFAULT VALUES
C
      DATA IRANDM/1/
      DATA NRF/20/
      DATA SIGMA/20*0.03/
      DATA ALAST/-99.9/
C
      READ(5, DATUM)
C
      READ 11,IDENT
   11 FORMAT(20A4)
      PUNCH 11,IDENT
C
      PRINT 16,IDENT
   16 FORMAT('1',20A4,/)
      PRINT 13
   13 FORMAT('0',20X,'INPUT DATA'//,10X,'N',10X,'FREQUENCY',8X,'R')
      DO 10 I=1,NRF
      READ 12,FREQ(I),R(I)
   12 FORMAT(F7.2,3X,2F10.3)
      PRINT 14,I,FREQ(I),R(I)
   14 FORMAT(' ',5X,I5,5X,F10.3,5X,2F10.3)
   10 CONTINUE
C
      READ 55,FINAL
   55 FORMAT(F7.0)
      IF(FINAL .LT. 0.0) GO TO 56
      PRINT 57
   57 FORMAT(' ','LAST CARD IS NOT NEGATIVE')
      GO TO 58
C
   56 CONTINUE
      DO 20 I=1,NRF
      PARTS=R(I)
      CALL RANDOM(IRANDM,X)
      RDMR(I)=X
      IRDMR(I)=IRANDM
      PART(1)=PART(1)+SIGMA(I)*X
      CALL RANDOM(IRANDM,X)
      RDMI(I)=X
      IRDMI(I)=IRANDM
      PART(2)=PART(2)+SIGMA(I)*X
   20 RR(I)=PARTS
      PRINT 16,IDENT
      PRINT 26
```

PROGRAM RANDOM (CONT)

```
      26 FORMAT('0',20X,'GAUSSIAN RANDOM X DATA')
         PRINT 27
      27 FORMAT('0',4X,'I',2X,'IRANDM',10X,'XREAL',10X,'IRANDM',
     *10X,'XIMAG'/)
         DO 28 I=1,NRF
      28 PRINT 29,I,IRDMR(I),RDMR(I),IRDMI(I),RDMI(I)
      29 FORMAT(' ',I5,1X,I10,2X,F10.3,3X,I10,5X,F10.3)
C
         PRINT 16,IDENT
         PRINT 21
      21 FORMAT('0',20X,'GAUSSIAN RANDOM R DATA')
         PRINT 22
      22 FORMAT('0',5X,'N',5X,'FREQUENCY',10X,'R',20X,'RR',20X,'SIGMA'/)
         DO 30 I=1,NRF
         PRINT 31,I,FREQ(I),R(I),RR(I),SIGMA(I)
      31 FORMAT(' ',2X,I5,F13.3,2X,2F10.3,2X,2F10.3,2X,F10.3)
         PUNCH 32,FREQ(I),RR(I),SIGMA(I)
      32 FORMAT(F7.2,3X,2F10.3,F10.3)
      30 CONTINUE
C
         PUNCH 32,ALAST
      58 CONTINUE
         STOP
         END
```

```
      SUBROUTINE RANDOM (IX,X)
C  THIS ROUTINE IS SPECIFIC TO SYSTEM 1110. IT GENERATES APPROXIMATELY
C                GAUSSIAN DISTRIBUTED RANDOM FLOATING NUMBERS WITH
C                UNIT STANDARD DEVIATION.  (2**29)/12 NUMBERS WILL BE
C                GENERATED BEFORE SEQUENCE IS REPEATED.
C  INITIAL VALUE OF IX MAY BE ANY      INTEGER OF NINE DIGITS OR LESS.
C  P IS A UNIFORMLY DISTRIBUTED RANDOM VARIABLE WITH VALUES
C                BETWEEN 0.0 AND 1.0.
C
C        RANDOM GENERATES A RANDOM NUMBER FROM A SEED AND SIMULATES AN
C         IBM 360 32 BIT WORD GENERATION.  THE CALLING SEQUENCE IS THE
C         SAME AS FOR THE IBM 360 RANDOM SUBROUTINE.
C         THE SEED 'IX' MUST BE A POSITIVE INTEGER.
      DATA IONES /017/
      IX=IX*2-1
      X= -6.0
      DO 11 L=1,12
      IX = IX * 65539
C     IF (IX) 5, 6, 6
      IF(BITS(IX,5,1) .EQ. 0) GO TO 6
    5 BITS (IX, 1, 4) = IONES
C     IX = IX + 2147483647 + 1
      IX = IX + 2147483647
      GO TO 7
    6 BITS (IX, 1, 4) = 0
    7 YFL = IX
      YFL = YFL * .4656613E-9
   11 X = X + YFL
      RETURN
      END
```

APPENDIX C

FORTRAN LISTING OF THE "INVERT" COMPUTER PROGRAM

PROGRAM INVERT

```
MORFITT*INVERT(1).MAIN
      1       C                              MAIN
      2       C          PROGRAM '''INVERT'''
      3       C          THIS VERSION OF THE IONOSPHERIC PROFILE INVERSION
      4       C          PROGRAM WILL WORK FOR  AN ARBITRARY CHOICE OF FREQUENCIES
      5       C          AS LONG AS THE FOLLOWING RULE IS APPLIED.
      6       C
      7       C                  ((FMAX-FMIN)/DELF)  .LE. 200.
      8       C
      9       C
     10              COMMON/NR COM/NR
     11              COMMON/PLOT C/IPROFP,IRPLOT
     12              COMMON/ANS COM/ANSWER(51)
     13              COMMON/NORG C/NORG
     14              COMMON/MIN C/MIN
     15              COMMON/DOPT C/DOPT
     16              COMMON/ROTAT C/ROTATE
     17              COMMON/DELF C/DELF
     18              COMMON/DLTAH C/DELTAH
     19              COMMON/HS COM/HTSTOP(2)
     20              COMMON/STOPS C/STOPS(2)
     21              COMMON/EXPNU C/COEFNU,EXPNU
     22              COMMON/IPA COM/IPRNTA
     23              COMMON/IPE COM/IPRNTE
     24              COMMON/ALPHA C/ALPHA(51)
     25              COMMON/NS COM/NR STPS(25,20)
     26
     27              COMMON/AL COM/ALAMDA
     28              COMMON/DL COM/DELTAL
     29              NAMELIST/DATUM/
     30             $           MIN,
     31             $           NR,
     32             $           DOPT,
     33             $           ROTATE,
     34             $           DELF,
     35             $           DELTAH,
     36             $           HTSTOP,
     37             $           STOPS,
     38             $           COEFNU,EXPNU,NUFLAG,
     39             $           IPRNTA,
     40             $           IPRNTE,
     41             $           ALPHA,
     42             $           NR STPS,
     43             $
     44             $           ALAMDA,
     45             $           DELTAL
     46              DIMENSION IBCD(20)
     47              DATA NAME/'NAME'/,ADAT/' &DA'/,DAT2/'TUM '/,
     48             $ IEND/' &EN'/,END2/'D   '/
     49       C
     50       C
     51              PRINT 100
```

```
  100 FORMAT('1')
C
      READ 901,IBCD
      PRINT 902,IBCD
C
      CALL D INPUT
      CALL FW INPT
C
      IF(IPROFP .NE. 1) GO TO 66
C
C        READ IN AN ORIGINAL PROFILE
C
      READ 901,IBCD
      PRINT 902,IBCD
      J=1
   60 READ 67,HTD,ELD
   67 FORMAT(F7.2,5X,1PE9.2)
      IF(HTD .LT. 0.0) GO TO 98
      ANSWER(J) = ALOG(ELD)
      J = J+1
      GO TO 60
   98 NORG=J-1
   66 CONTINUE
C
      MIN = 10
      DOPT = 1.0
      NR=1

      DELF= 0.1
      DELTAH = 1.0
      HTSTOP(1) = 93.0
      HTSTOP(2) = 50.0
      STOPS(1) = 1.0E3
      STOPS(2) = 3.0E0
      COEFNU = 1.816E11
      EXPNU = -0.15
      NUFLAG = 0
      IPRNTA = 0
      IPRNTE = 0
      ROTATE = 0.0
      ALAMDA = 0.0
      DELTAL = 1.0/64.0
C
      READ 901,IBCD
      PRINT 902,IBCD
C
      IF(IBCD(1) .EQ. NAME) GO TO 90
      IF(IBCD(1) .NE. NAME) PRINT 600
  600 FORMAT('0',' NO NAMELIST INPUT.---PROGRAM STOPS---')
      GO TO 999
   90 READ(5,DATUM)
      PRINT 601,ADAT,DAT2
```

```
104          601 FORMAT(' ',2A4)
105              IF (NUFLAG .EQ. 0) GO TO 27
106              COEFNU = 4.303E11
107              EXPNU = -0.1622
108           27 CONTINUE
109              PRINT 602,HTSTOP(1),HTSTOP(2),STOPS(1),STOPS(2)
110          602 FORMAT(' ',' HTSTOP(1) = ',F8.3,' HTSTOP(2) = ',F8.3,
111             * ' STOPS(1) = ',1PE10.3,' STOPS(2) = ',1PE10.3)
112              PRINT 201,MIN,DOPT,        DELTAH
113          201 FORMAT(' ',' MIN = ',I5,   ' DOPT = ',F9.3,
114             $ ' DELTAH = ',F9.3)
115              PRINT 202, ROTATE ,ALAMDA,DELTAL,IPRNTA,IPRNTE,NUFLAG
116          202  FORMAT(' ', ROTATE    = ',F9.3,' ALAMDA = ',F9.3,' DELTAL = ',
117             $ F9.4,' IPRNTA = ',I2,' IPRNTE = ',I2,' NUFLAG = ',I2)
118              IF(NUFLAG .EQ. 1) PRINT 203,NUFLAG,COEFNU,EXPNU
119          203 FORMAT(' ',' NUFLAG = ',I2,' COEFNU = ',1PE10.3,
120             $ ' EXPNU = ',0PF8.4)
121              PRINT 204,IEND,END2
122          204 FORMAT(' ', 2A4,//)
123          C
124              CALL CONTRL
125          C
126            901 FORMAT (20A4)
127            902 FORMAT (' ',20A4)
128          C
129            999 STOP
130          C
131              END
```

```
@PRT,S INVERT.CONTRL
```

```
MORFITT*INVERT(1).CONTRL
      1              SUBROUTINE CONTRL
      2      C
      3              COMMON/NR COM/NR
      4              COMMON/PLOT C/IPROFP,IRPLOT
      5              COMMON/MIN C/MIN
      6              COMMON/D OPT C/D OPT
      7              COMMON/NS COM/NR STPS(25,20)
      8              COMMON/NRF COM/NR F
      9              COMMON/NRA COM/NR A
     10              COMMON/ALPHA C/ALPHA(51)
     11              COMMON/DA COM/DA(51)
     12              COMMON/ADIST C/ADIST
     13              COMMON/OVRFLO/IOVFLO
     14              COMMON/S COM/S
     15              COMMON/AL COM/ALAMDA
     16              COMMON/SN COM/SNEXT
     17              COMMON/DL COM/DELTAL
     18              DIMENSION ALPHA0(51),THRESH(4),NS0(25,20)
     19              DATA THRESH/0.8,1.2,2.0,4.0/
     20      C
     21      C
     22              NSEC = 60*MIN
     23                 FNSEC = NSEC
     24                 CALL SETCLK
     25      C
     26              IFIRST = 1
     27      C
     28              CALL SET HTS
     29              CALL INTERP
     30              CALL LOGS
     31      C
     32              PRINT 101
     33        101 FORMAT('0','END OF INITILIZATION')
     34              PRINT 103
     35        103 FORMAT('1','INITIAL PROFILE--')
     36              IF(ALAMDA .EQ. 0.0 ) GO TO 51
     37              PRINT 104,NR,ALAMDA
     38        104 FORMAT(' ',3X,'CONTINUE WITH LAMBDA',I3,' = ',F12.5,1X,
     39           $ '(NOT USED UNTIL SETTLE-DOWN)')
     40      C
     41         15 CALL PRINT N
     42              PRINT 105,NR
     43        105 FORMAT(' ',3X,'FULL-WAVE SOLUTIONS',I3)
     44              CALL R DRDA
     45              IF (IOVFLO .NE. 0) GO TO 90
     46              CALL PRNT RS
     47              PRINT 108,NR,S
     48        108 FORMAT('0',3X,'S',I3,' = ',F9.3,1X,'USING FULL-WAVE VALUES')
     49      C
     50              IF(IRPLOT .EQ. 1) CALL RPLOTS
     51      C
```

```
52          20 PRINT 200
53         200 FORMAT('1','SETTLE-DOWN---')
54             NR = NR + 1
55       C
56             CALL SOL EQS (0)
57       C
58             RATIO = ADIST/D OPT
59             PRINT 202,NR,ALAMDA,SNEXT,ADIST,RATIO,NR
60         202 FORMAT(' ',3X,'SOLVE EQS WITH LAMBDA',I3,' = ',F12.5,1X,
61            $ 'GIVING S PRIME = ',F9.3,2X, 'D PRIME = ',F7.3,2X,'RATIO = ',
62            $ F8.2,1X,'AND SETTLE PROFILE',I3)
63             CALL PRINT P
64       C
65             IF (RATIO .LT. THRESH(1)) DELTAL = SQRT(2.0)*DELTAL
66             IF (RATIO .GT. THRESH(2)) DELTAL = DELTAL/2.0
67       C
68             IF (RATIO .LT. THRESH(4) .OR. IFIRST .NE. 0) GO TO 40
69          30 CONTINUE
70             DO 32 J=1,NR A
71          32 ALPHA(J) = ALPHAO(J)
72             ALAMDA = ALO
73             PRINT 300,NR,ALANDA,NR,DELTAL
74         300 FORMAT ('1','BACKSTEP---',
75            $ 15X,'SAVED LAMBDA',I3,' = ',F12.5,' DELTA LAMBDA',I3,' = ',
76            $ F12.5,' (NOT USED)'/)
77             NR JHF = (NR A-1)/2
78             DO 39 L=1,NR F
79             DO 39 JHF=1,NR JHF
80          39 NR STPS(JHF,L) = NSO(JHF,L)
81                CALL LKCLKS(SKEY)
82                IF(FNSEC .LT. SKEY) GO TO 70
83             GO TO 15
84       C
85          40 IF (RATIO .LT. THRESH(3)) GO TO 50
86             NR = NR +1
87             PRINT 400,NR,ALAMDA,NR,DELTAL
88         400 FORMAT('1','SAME LAMBDA---',
89            $ 15X,'(SAME) LAMBDA',I3,' = ',F12.5,9X,
90            $ 'DELTA LAMBDA',I3,' = ',F12.5,' (NOT USED)'/,19X,
91            *'PREVIOUS SOLUTION FROM SETTLE DOWN IS USED')
92             CALL LKCLKS(SKEY)
93             IF(FNSEC .LT. SKEY) GO TO 70
94             GO TO 60
95       C
96          50 NR = NR + 1
97             PRINT 500,NR,DELTAL
98         500 FORMAT('1','NEW LAMBDA---',15X,'DELTA LAMBDA',I3,' = ',F12.5)
99             IFIRST=0
100      C
101            ALO = ALAMDA
102            ALAMDA = ALAMDA+DELTAL
103      C
```

```
      51 CALL SOL EQS (1)
         PRINT 504,NR,ALAMDA,ADIST,SNEXT,NR
      504 FORMAT(' ',3X,'SOLVE EQS WITH LAMBDA',I3,' = ',F12.5,1X,
     $ 'GIVING D = ',F7.3,2X,'LINEAR S = ',F9.3,1X,'AND PROFILE',I3)
C
            CALL LKCLKS(SKEY)
            IF(FNSEC .LT. SKEY) GO TO 70
C
            DO 52 J=1,NR A
      52 ALPHAO(J) = ALPHA(J)
            NR JHF = (NR A-1)/2
            DO 58 L=1,NR F
            DO 58 JHF=1,NR JHF
      58 NSO(JHF,L) = NR STPS(JHF,L)
C
      60 DO 61 J=1,NR A
      61 ALPHA(J) = ALPHA(J)+DA(J)
C
            CALL PRINT N
            PRINT 601,NR
      601 FORMAT(' ',3X,'FULL-WAVE SOLUTIONS',I3)
            CALL R DRDA
            IF(IOVFLO .NE. 0) GO TO 90
            CALL PRNT RS
            PRINT 602,NR,S
      602 FORMAT('0',3X,'S',I3,' = ',F9.3,1X,'USING FULL-WAVE VALUES')
C
            IF(IRPLOT .EQ. 1) CALL RPLOTS
C
            GO TO 20
C
      70 ALAMDA = ALO
            CALL CARDS
            STOP
C
      90 PRINT 990
      990 FORMAT ('0','IOVFLO FLAG SET')
            STOP
C
            END

@PRT,S INVERT.SETHTS
```

```
MORFITT*INVERT(1).SETHTS
      1         SUBROUTINE SET HTS
      2   C
      3         COMMON/DLTAH C/DELTAH
      4         COMMON/HTS COM/HTS(51)
      5         COMMON/HS COM/HTSTOP(2)
      6         COMMON/STOPS C/STOPS(2)
      7         COMMON/NRF COM/NR F
      8         COMMON/NS COM/NR STPS(25,20)
      9         COMMON/ALPHA C/ALPHA(51)
     10         COMMON/AL COM/ALAMDA
     11         COMMON/NRA COM/NR A
     12         COMMON/ALIMS C/AMAX,AMIN
     13   C
     14         DATA EPS/0.001/
     15   C
     16   C
     17         AMAX = ALOG(STOPS(1))
     18         AMIN = ALOG(STOPS(2))
     19   C
     20         FLOTE = (HTSTOP(1)-HTSTOP(2))/(2.0*DELTAH)
     21         NR JHF = FLOTE+0.01
     22         SLOP = ABS(FLOTE-NR JHF)
     23         IF (SLOP .GT. 0.02) GO TO 90
     24   C
     25         IF (ALAMDA .EQ. 0.0) ALPHA(1) = (AMAX+AMIN)/2.0
     26         HTS(1) = HTSTOP(1)
     27         J = 1
     28      21 J = J+1
     29         HTS(J) = HTS(J-1)-DELTAH
     30         IF (ALAMDA .EQ. 0.0) ALPHA(J) = ALPHA(1)
     31         IF(HTS(J)-0.01 .GT. HTSTOP(2) ) GO TO 21
     32         NR A = J
     33   C
     34         IF (NR A-1 .NE. 2*NR JHF) GO TO 90
     35         IF (ALAMDA .NE. 0.0) RETURN
     36   C
     37         DO 31 L=1,NR F
     38         DO 31 JHF=1,NR JHF
     39      31 NR STPS(JHF,L) = 2
     40         RETURN
     41   C
     42      90 PRINT 900
     43     900 FORMAT ('0','(HTSTOP(1)-HTSTOP(2))/DELTAH MUST BE A MULTIPLE OF 2.
    $0')
     44         STOP
     45   C
     46         END
     47

&PRT,S INVERT.CARDS
```

```
MORFITT*INVERT(1).CARDS
      1              SUBROUTINE CARDS
      2        C
      3              COMMON/ALPHA C/ALPHA(51)
      4              COMMON/NR COM/NR
      5              COMMON/NS COM/NR STPS(25,20)
      6              COMMON/ROTAT C/ROTATE
      7              COMMON/AL COM/ALAMDA
      8              COMMON/DL COM/DELTAL
      9              COMMON/NRA COM/NR A
     10              COMMON/NRF COM/NR F
     11        C
     12        C
     13              PRINT 100
     14          100 FORMAT ('1','OUTPUT CARDS--')
     15        C
     16              PRINT 201,(ALPHA(J),J=1,NR A)
     17          201 FORMAT(' ',' ALPHA = ', 10(F6.3,','),/,(' ',10X,10(F6.3,',')))
     18              PUNCH 202,(ALPHA(J),J=1,NR A)
     19          202 FORMAT(     ' ALPHA = ', 10(F6.3,','),/,     (10X,10(F6.3,',')))
     20        C
     21              NR JHF = (NR A-1)/2
     22              DO 31 L=1,NR F
     23              PRINT 301,L,(NR STPS(JHF,L),JHF =1,NR JHF)
     24          301 FORMAT (' ',' NRSTPS(1,',I2,') = ',12(I4,','),/,(' ',16X,12(I4,','
     25             $)))
     26           31 PUNCH 302,L,(NR STPS(JHF,L),JHF =1,NR JHF)
     27          302 FORMAT (     ' NRSTPS(1,',I2,') = ',12(I4,','),/,(    16X,12(I4,','
     28             $)))
     29        C
     30              PRINT 401, ROTATE
     31          401 FORMAT (' ',' ROTATE = ',F6.1,',')
     32              PUNCH 402, ROTATE
     33          402 FORMAT (' ROTATE  = ',F6.1,',')
     34        C
     35              PRINT 501,ALAMDA
     36          501 FORMAT (' ',' ALAMDA = ',1PE9.2,',')
     37              PUNCH 502,ALAMDA
     38          502 FORMAT (' ALAMDA = ',1PE9.2,',')
     39        C
     40              PRINT 601,DELTAL
     41          601 FORMAT (' ',' DELTAL = ',F12.5,',')
     42              PUNCH 602,DELTAL
     43          602 FORMAT (     ' DELTAL = ',F12.5,',')
     44        C
     45              PRINT 701,NR
     46          701 FORMAT(' ',' NR = ',I3,',')
     47              PUNCH 702,NR
     48          702 FORMAT( ' NR = ',I3,',')
     49        C
     50        C             ROUTINE ADDED TO CHECK FOR OVERFLOW
     51          610 CALL OVCHK('CARDS ',610)
```

```
      52                RETURN
      53          C
      54                END

@PRT,S INVERT.SOLEQS
```

```
MORFITT*INVERT(1).SOLEQS
      1              SUBROUTINE SOL EQS (LFLAG)
      2        C
      3              COMMON/DLTAH C/DELTAH
      4              COMMON/IPA COM/IPRNTA
      5              COMMON/ALIMS C/AMAX,AMIN
      6              COMMON/ALPHA C/ALPHA(51)
      7              COMMON/HTS COM/HTS(51)
      8              COMMON/NRA COM/NR A
      9              COMMON/NRF COM/NR F
     10              COMMON/AL COM/ALAMDA
     11              COMMON/RNORM C/R NORM(40)
     12              COMMON/DRDA C/DRDA(40,51)
     13              COMMON/S COM/S
     14              COMMON/DA COM/DA(51)
     15              COMMON/SN COM/SNEXT
     16              COMMON/ADIST C/ADIST
     17              COMMON/AU COM/ALP UNC(51)
     18              COMMON/RSN COM/RESLTN(51)
     19              COMMON/TEMP C/B(1326),A(2040)
     20              DIMENSION PEN(2),CAP A(51)
     21              REAL*8 B,A,V(51),AUX(51),DSUM
     22              DATA EPS/1.0E-4/
     23              DATA ALN TEN/2.302585/
     24        C
     25        C
     26              NR M = 2*NR F
     27              IF(ALAMDA .EQ. 0.0 .AND. LFLAG .NE. 0) GO TO 12
     28              S = 0.0
     29              DO 11 I=1,NR M
     30           11 S = S+R NORM(I)**2
     31           12 CONTINUE
     32        C
     33              FACTR1 = 1.0
     34              IF (ALAMDA .NE. 0.0) FACTR1 = 1.0E5/DELTAH**3/ALAMDA**2
     35              PENLTY = FACTR1*1.0E2
     36              PEN(1) = 0.0
     37              IF (ALPHA(1) .LE. AMAX) PEN(1) = PENLTY
     38              PEN(2) = 0.0
     39              IF (ALPHA(NR A) .GE. AMIN) PEN(2) = PENLTY
     40              ICOUNT = 0
     41        C
     42           20 JJ J = 1
     43              DO 23 J=1,NR A
     44              DO 23 JJ=1,J
     45              DSUM = 0.0
     46              IF (ALAMDA .EQ. 0.0) GO TO 22
     47              DO 21 I=1,NR M
     48           21 DSUM = DSUM+DRDA(I,JJ)*DRDA(I,J)
     49           22 B(JJ J) = DSUM
     50           23 JJ J = JJ J+1
     51        C
```

```
      DO 25 J=1,NR A
      DSUM = 0.0
      IF (ALAMDA .EQ. 0.0) GO TO 25
      DO 24 I=1,NR M
   24 DSUM = DSUM+DRDA(I,J)*R NORM(I)
   25 V(J) = -DSUM
C
      B(1) = B(1)+2.0*FACTR1
      B(2) = B(2)-3.0*FACTR1
      B(3) = B(3)+6.0*FACTR1
      J J = 6
      NRAM2 = NR A-2
      DO 27 J=3,NRAM2
      B(J J-2) = B(J J-2)+FACTR1
      B(J J-1) = B(J J-1)-4.0*FACTR1
      B(J J) = B(J J)+6.0*FACTR1
   27 J J = J J+J+1
      B(J J-2) = B(J J-2)+FACTR1
      B(J J-1) = B(J J-1)-4.0*FACTR1
      B(J J) = B(J J)+5.0*FACTR1
      J J = J J+NR A
      B(J J-2) = B(J J-2)+FACTR1
      B(J J-1) = B(J J-1)-2.0*FACTR1
      B(J J) = B(J J)+FACTR1
C
      V(1) = V(1)-FACTR1*(2.0*ALPHA(1)-3.0*ALPHA(2)+ALPHA(3))
      V(2) = V(2)-FACTR1*(-3.0*ALPHA(1)+6.0*ALPHA(2)-4.0*ALPHA(3)
     $           +ALPHA(4))
      NRAM2 = NR A-2
      DO 28 J=3,NRAM2
   28 V(J) = V(J)-FACTR1*(ALPHA(J-2)-4.0*ALPHA(J-1)+6.0*ALPHA(J)
     $           -4.0*ALPHA(J+1)+ALPHA(J+2))
      V(NR A-1) = V(NR A-1)-FACTR1*(ALPHA(NR A-3)-4.0*ALPHA(NR A-2)
     $           +5.0*ALPHA(NR A-1)-2.0*ALPHA(NR A))
      V(NR A) = V(NR A)-FACTR1*(ALPHA(NR A-2)-2.0*ALPHA(NR A-1)
     $           +ALPHA(NR A))
C
      B(1) = B(1)+PEN(1)
      J J = (NR A*(NR A+1))/2
      B(J J) = B(J J)+PEN(2)
C
      V(1) = V(1)-PEN(1)*(ALPHA(1)-AMAX)
      V(NR A) = V(NR A)-PEN(2)*(ALPHA(NR A)-AMIN)
C
      CALL DGELS (V,B,NR A,1,EPS,IER,AUX)
      IF (IER .LT. 0) PRINT 903,IER
  903 FORMAT(' ','FOR DA SOLUTION, IER =',I3)
C
      DO 31 J=1,NR A
   31 DA(J) = V(J)
C
      IAGAIN = 0
```

```
      IF (PEN(1) .EQ. 0.0 .OR. ALPHA(1)+DA(1) .LE. AMAX) GO TO 32
      PEN(1) = 0.0
      IAGAIN = 1
      GO TO 33
   32 IF (PEN(1) .NE. 0.0 .OR. ALPHA(1)+DA(1) .GT. AMAX) GO TO 33
      PEN(1) = PENLTY
      IAGAIN = 1
   33 IF (PEN(2) .EQ. 0.0 .OR. ALPHA(NR A)+DA(NR A) .GE. AMIN) GO TO 34
      PEN(2) = 0.0
      IAGAIN = 1
      GO TO 35
   34 IF (PEN(2) .NE. 0.0 .OR. ALPHA(NR A)+DA(NR A) .LT. AMIN) GO TO 35
      PEN(2) = PENLTY
      IAGAIN = 1
   35 IF (IAGAIN .EQ. 0) GO TO 40
      ICOUNT = ICOUNT+1
      IF (ICOUNT .LT. 5) GO TO 20
      PRINT 305
  305 FORMAT ('0','ICOUNT TOO LARGE')
      STOP
C
   40 SNEXT = 0.0
      ADIST=0.0
      IF(ALAMDA .EQ. 0.0 .AND. LFLAG .NE. 0) RETURN
      DO 46 I=1,NR M
      DSUM = 0.0
      DO 45 J=1,NR A
   45 DSUM = DSUM+DRDA(I,J)*DA(J)
      RNEXT = R NORM(I)+DSUM
   46 SNEXT = SNEXT+RNEXT**2
C
      SUM SQ = 0.0
      DO 47 J=1,NR A
      IF (ALPHA(J) .GT. AMIN .AND. ALPHA(J) .LT. AMAX)
     $          SUM SQ = SUM SQ+DA(J)**2
   47 CONTINUE
      ADIST = SQRT(SUM SQ*DELTAH)/ALN TEN
C
      IF (LFLAG .NE. 0) RETURN
      IF (ALAMDA .EQ. 0.0) GO TO 70
      JJ J = 1
      DO 52 J=1,NR A
      DO 52 JJ=1,J
      DSUM = 0.0
      DO 51 I=1,NR M
   51 DSUM = DSUM+DRDA(I,JJ)*DRDA(I,J)
      B(JJ J) = DSUM
   52 JJ J = JJ J+1
C
      B(1) = B(1)+1.0*FACTR1
      B(2) = B(2)-2.0*FACTR1
      B(3) = B(3)+5.0*FACTR1
```

```
156             J J = 0
157             NRAM2 = NR A-2
158             DO 53 J=3,NRAM2
159             B(J J-2) = B(J J-2)+FACTR1
160             B(J J-1) = B(J J-1)-4.0*FACTR1
161             B(J J) = B(J J)+6.0*FACTR1
162          53 J J = J J+J+1
163             B(J J-2) = B(J J-2)+FACTR1
164             B(J J-1) = B(J J-1)-4.0*FACTR1
165             B(J J) = B(J J)+5.0*FACTR1
166             J J = J J+NR A
167             B(J J-2) = B(J J-2)+FACTR1
168             B(J J-1) = B(J J-1)-2.0*FACTR1
169             B(J J) = B(J J)+FACTR1
170       C
171             J I = 1
172             DO 57 I=1,NR M
173             DO 57 J=1,NR A
174             A(J I) = DRDA(I,J)
175          57 J I = J I+1
176       C
177             CALL DGELS (A,B,NR A,NR M,EPS,IER,AUX)
178             IF (IER .LT. 0) PRINT 905,IER
179         905 FORMAT(' ','FOR A SOLUTION, IER = ',I3)
180       C
181             DO 62 J=1,NR A
182             DSUM = 0.0
183             J I = J
184             DO 61 I=1,NR M
185             DSUM = DSUM+A(J I)**2
186          61 J I = J I+NR A
187          62 ALP UNC(J) = DSQRT(DSUM)/ALN TEN
188       C
189             DO 66 J=1,NR A
190             DO 64 JJ=1,NR A
191             DSUM = 0.0
192             J I = J
193             DO 63 I=1,NR M
194             DSUM = DSUM+A(J I)*DRDA(I,JJ)
195          63 J I = J I+NR A
196          64 CAP A(JJ) = DSUM
197       C
198             IF (IPRNTA .NE. 0) PRINT 907,J,HTS(J),(CAP A(JJ),JJ=1,NR A)
199         907 FORMAT(' ','CAP A',I4,' AT',F7.2,' = ',/,(10X,10(1PE11.2)))
200       C
201             SUM NUM = 0.0
202             SUM DEN = 0.0
203             DO 65 JJ=1,NR A
204             SUM NUM = SUM NUM+(HTS(J)-HTS(JJ))**2*ABS(CAP A(JJ))
205          65 SUM DEN = SUM DEN+ABS(CAP A(JJ))
206          66 RESLTN(J) = SQRT(SUM NUM/SUM DEN)
207             RETURN
```

```
      C
      70 DO 71 J=1,NR A
         ALP UNC(J) = 0.0
      71 RESLTN(J) = 0.0
         RETURN
      C
         END
```

```
@PRT,S INVERT.PRINTN
```

```
MORFITT*INVERT(1).PRINTN
      1         SUBROUTINE PRINT N
      2   C
      3         COMMON/NORG C/NORG
      4         COMMON/ANS COM/ANSWER(51)
      5         COMMON/PLOT C/IPROFP,IRPLOT
      6         COMMON/NRA COM/NR A
      7         COMMON/HTS COM/HTS(51)
      8         COMMON/ALPHA C/ALPHA(51)
      9         COMMON/AU COM/ALP UNC(51)
     10         COMMON/RSN COM/RESLTN(51)
     11         COMMON/DA COM/DA(51)
     12         COMMON/ALIMS C/AMAX,AMIN
     13         INTEGER V
     14         DIMENSION LINE(81)
     15   C
     16   C
     17         DATA ISYMO/'*'/,ISYMN/'+'/,IBLANK/' '/,ISYMS/'$'/
     18         DATA ALN TEN/2.302585/
     19         DATA V/01740000000000/
     20   C
     21         FMESH=4.0*ALN TEN/80.0
     22   C
     23         IF(IPROFP .EQ. 1 .AND. NRA .NE. NORG) GO TO 988
     24         PRINT 100,V,V,V,V,V
     25     100 FORMAT ('0',5X,'HT',9X,'N',18X, A1,19('-'), A1,19('-'),
     26        *  A1,19('-'), A1,19('-'), A1)
     27         DO 11 J=1,NR A
     28         EN = EXP(ALPHA(J))
     29         DO 50 K = 1,81
     30      50 LINE(K) = IBLANK
     31         IF(IPROFP .EQ. 0) GO TO 60
     32         JPOST=ANSWER(J)/FMESH+1.5
     33         IF(JPOST .GE. 1 .AND. JPOST .LE. 81)
     34        *LINE(JPOST)=ISYMO
     35      60 CONTINUE
     36         JPOST=ALPHA(J)/FMESH+1.5
     37         IF(JPOST .GE. 1 .AND. JPOST .LE. 81)
     38        *LINE(JPOST)=ISYMN
     39         IF(J .NE. 1) GO TO 67
     40         JPOST=AMAX/FMESH+1.5
     41         LINE(JPOST)=ISYMS
     42      67 IF(J .NE. NR A) GO TO 11
     43         JPOST=AMIN/FMESH+1.5
     44         LINE(JPOST)=ISYMS
     45      11 PRINT 105,HTS(J),EN,V,LINE,V
     46     105 FORMAT (' ',F7.2,1X,1PE9.2,17X, A1,81A1, A1)
     47         GO TO 500
     48   C
     49   C
     50         ENTRY PRINT P
     51         PRINT 200,V,V,V,V,V
```

```
      200 FORMAT ('0',5X,'HT',9X,'N',3X,'UNC',4X,'RES',5X, A1,19('-'),
     *  A1,19('-'), A1,19('-'), A1,19('-'), A1)
          DO 21 J=1,NR A
          EN = EXP(ALPHA(J)+DA(J))
          DO 70 K=1,81
       70 LINE(K)=IBLANK
          IF(IPROFP .EQ. 0) GO TO 61
          JPOST = ANSWER(J)/FMESH+1.5
          IF(JPOST .GE. 1 .AND. JPOST .LE. 81)
         *LINE(JPOST) = ISYMO
       61 CONTINUE
          JPOST=(ALPHA(J)+DA(J))/FMESH+1.5
          IF(JPOST .GE. 1 .AND. JPOST .LE. 81)
         *LINE(JPOST)=ISYMN
          IF(J .NE. 1) GO TO 68
          JPOST=AMAX/FMESH+1.5
          LINE(JPOST)=ISYMS
       68 IF(J .NE. NR A) GO TO 21
          JPOST=AMIN/FMESH+1.5
          LINE(JPOST)=ISYMS
       21 PRINT 205,HTS(J),EN,ALP UNC(J),RESLTN(J),V,LINE,V
      205 FORMAT (' ',F7.2,1X,1PE9.2,0PF6.2,1X,0PF6.2,4X, A1,81A1, A1)
C
      500 PRINT 501,V,V,V,V,V
      501 FORMAT(' ',35X, A1,19('-'), A1,19('-'), A1,19('-'), A1,
     *  19('-'), A1 )
          PRINT 502
      502 FORMAT(' ',35X,'0',19X,'1',19X,'2',19X,'3',19X,'4')
C
          GO TO 800
C
      988 PRINT 987,NR A,NORG
      987 FORMAT('0',//,'PROGRAN STOPS***',' NRA = ',I5,' AND NORG = ',I5)
          STOP
      800 CONTINUE
          RETURN
C
          END

@PRT,S INVERT.RPLOTS
```

```
MORFITT*INVERT(1).RPLOTS
      1         SUBROUTINE RPLOTS
      2   C
      3         COMMON/NRF COM/NR F
      4         COMMON/RDATA C/R DATA(2,20)
      5         COMMON/DSPLA C/R DSPLA(6,20)
      6         INTEGER V
      7         DIMENSION ISYMD(20),ISYMC(20),XD(20),YD(20),XC(20),YC(20),
      8        * LINE(101)
      9   C
     10         DATA ISYMD/'A','B','C','D','E','F','G','H','I','J',
     11        $           'K','L','M','N','O','P','Q','R','S','T'/
     12         DATA ISYMC/'1','2','3','4','5','6','7','8','9','0',
     13        $           '1','2','3','4','5','6','7','8','9','0'/
     14         DATA IBLANK/' '/
     15         DATA V/0174000000000/
     16   C
     17         XMAX=-1.0E9
     18         XMIN=+1.0E9
     19         YMAX=-1.0E9
     20         YMIN=+1.0E9
     21   C
     22         DO 10 L=1,NR F
     23         XD(L)=R DATA(1,L)
     24         YD(L)=R DATA(2,L)
     25         IF(XD(L) .LT. XMIN) XMIN=XD(L)
     26         IF(XD(L) .GT. XMAX) XMAX=XD(L)
     27         IF(YD(L) .LT. YMIN) YMIN=YD(L)
     28         IF(YD(L) .GT. YMAX) YMAX=YD(L)
     29      10 CONTINUE
     30   C
     31         DO 20 L=1,NR F
     32         XC(L)=R DSPLA(1,L)
     33         YC(L)=R DSPLA(2,L)
     34         IF(XC(L) .LT. XMIN) XMIN=XC(L)
     35         IF(XC(L) .GT. XMAX) XMAX=XC(L)
     36         IF(YC(L) .LT. YMIN) YMIN=YC(L)
     37         IF(YC(L) .GT. YMAX) YMAX=YC(L)
     38      20 CONTINUE
     39   C
     40         IF((XMAX-XMIN)/10.0 .GT. (YMAX-YMIN)/6.5) GO TO 100
     41         YMESH=(YMAX-YMIN)/52.0
     42         XMESH=YMESH*0.8
     43         XMAX=XMIN+(100.0*XMESH)
     44         GO TO 200
     45     100 XMESH=(XMAX-XMIN)/100.0
     46         YMESH=XMESH/0.8
     47         YMAX=YMIN+(52.0*YMESH)
     48     200 CONTINUE
     49         PRINT 30
     50      30 FORMAT('1',' PLOT OF R-VALUES',5X, 'DATA VALUES = ALPHAMERIC',
     51        * 5X,'COMPUTED VALUES = NUMERIC')
```

```
      YTOP=YMAX+YMESH
      PRINT 31,YTOP
   31 FORMAT(' ',F10.4,101('-') )
      DO 300 I=1,53
      DO 350 J=1,101
  350 LINE(J) = IBLANK
      DO 400 L=1,NR F
      ITEST = (YD(L)-YMIN)/YMESH+0.5
      IF((ITEST +I) .NE. 53) GO TO 400
      JPOST = (XD(L)-XMIN)/XMESH+1.5
      LINE(JPOST) = ISYMD(L)
  400 CONTINUE
C
      DO 401 L=1,NRF
      ITEST=(YC(L)-YMIN)/YMESH+0.5
      IF((ITEST+I) .NE. 53) GO TO 401
      JPOST=(XC(L)-XMIN)/XMESH+1.5
      LINE(JPOST) = ISYMC(L)
  401 CONTINUE
C
  300 PRINT 555,V,LINE,V
  555 FORMAT(' ',9X, A1,101A1, A1)
      YBOT=YMIN-YMESH
      PRINT 32,YBOT
   32 FORMAT(' ',F10.4,101('-') )
      PRINT 33,XMIN,XMAX
   33 FORMAT(' ',5X,F10.4,90X,F10.4)
C
      RETURN
      END

@PRT,S INVERT.DINPUT
```

```
MORFITT*INVERT(1).DINPUT
      1          SUBROUTINE D INPUT
      2  C
      3          COMMON/F COM/F(20)
      4          COMMON/RDATA C/R DATA(20)
      5          COMMON/SIGMA C/SIGMA(20)
      6          COMMON/NRF COM/NR F
      7          COMPLEX R DATA,R
      8          DIMENSION ID(20)
      9  C
     10  C
     11          READ 101,ID
     12    101   FORMAT (20A4)
     13          PRINT 102,ID
     14    102   FORMAT (' ',20A4)
     15  C
     16          L = 1
     17     11   READ 103,FREQ,R,SIG
     18    103   FORMAT (F7.2,3X,2F10.3,F10.3)
     19          PRINT 104,FREQ,R,SIG
     20    104   FORMAT (' ',F7.2,3X,2F10.3,F10.3)
     21          IF (FREQ .LT. 0.0) GO TO 15
     22          F(L) = FREQ
     23          R DATA(L) = R
     24          SIGMA(L) = SIG
     25          L = L+1
     26          GO TO 11
     27     15   NR F = L-1
     28          RETURN
     29  C
     30          END

@PRT,S INVERT.INTERP
```

```
MORFITT*INVERT(1).INTERP
      1                SUBROUTINE INTERP
      2        C
      3                COMMON/NRF COM/NR F
      4                COMMON/DELF C/DELF
      5                COMMON/F COM/F(20)
      6                COMMON/IF COM/INDEXF(20)
      7                COMMON/DIDR C/DIDR(205,20)
      8                COMMON/TEMP C/A(1000),V(4000)
      9                REAL*8 A,V
     10                DATA MAXNR/205/
     11        C
     12        C
     13                INDEXF(1)=3
     14                DO 11 L=2,NR F
     15           11   INDEXF(L) = 3.5+(F(L)-F(1))/DELF
     16                NR EQ = INDEXF(NR F)+2
     17                IF(NR EQ .GT. MAXNR) GO TO 90
     18        C
     19        C
     20                A(1) = 1.0
     21                A(2) = -2.0
     22                A(3) = 1.0
     23                A(4) = -2.0
     24                A(5) = 5.0
     25                A(6) = -4.0
     26                A(7) = 1.0
     27        C
     28                A(8) = 0.0
     29                A(9) = 0.0
     30                A(10) = 1.0
     31                A(11) = 0.0
     32                A(12) = 0.0
     33        C
     34                K = 15
     35                DO 23 L=2,NR F
     36           21   IF(K/5+1 .EQ. INDEXF(L)) GO TO 22
     37                A(K-2) = 1.0
     38                A(K-1) = -4.0
     39                A(K) = 6.0
     40                A(K+1) = -4.0
     41                A(K+2) = 1.0
     42                K = K+5
     43                GO TO 21
     44        C
     45           22   A(K-2) = 0.0
     46                A(K-1) = 0.0
     47                A(K) = 1.0
     48                A(K+1) = 0.0
     49                A(K+2) = 0.0
     50           23   K = K+5
     51        C
```

```
52              A(K-2) = 1.0
53              A(K-1) = -4.0
54              A(K) = 5.0
55              A(K+1) = -2.0
56              K = K+4
57              A(K-2) = 1.0
58              A(K-1) = -2.0
59              A(K) = 1.0
60        C
61              KL=1
62              DO 31 L=1,NR F
63              DO 31 K=1,NR EQ
64              V(KL)=0.0
65           31 KL=KL+1
66        C
67              DO 33 L=1,NR F
68              KL = (L-1)*NR EQ+INDEXF(L)
69           33 V(KL)=1.0
70        C
71              CALL DGELB (V,A,NR EQ,NR F,2,2,1.0E-4,IER)
72              PRINT 401,IER
73          401 FORMAT ('1','IN DGELB, IER = ',I3,/)
74        C
75              KLIMIT = NR EQ-1
76              DO 52 K=2,KLIMIT
77              KL=K
78              DO 51 L=1,NR F
79              DIDR(K,L)=V(KL+1)-V(KL-1)
80           51 KL=KL+NR EQ
81           52 PRINT 501,(DIDR(K,L),L=1,NR F)
82          501 FORMAT (' ',10F10.5)
83              RETURN
84        C
85           90 PRINT 900
86          900 FORMAT('0','DELF TOO SMALL FOR FREQUENCY RANGE USED')
87              STOP
88        C
89              END

&PRT,S INVERT.LOGS
```

```
MORFITT*INVERT(1).LOGS
      1                SUBROUTINE LOGS
      2        C
      3                COMMON/DPHAS C/DPHASE
      4                COMMON/F COM/F(20)
      5                COMMON/RDATA C/R DATA(20)
      6                COMMON/SIGMA C/SIGMA(20)
      7                COMMON/NRF COM/NR F
      8                COMMON/DIDR C/DIDR(205,20)
      9                COMMON/IF COM/INDEXF(20)
     10                COMMON/RD COM/RD(20)
     11                COMMON/OVSIG C/OVR SIG(20,20)
     12                COMMON/TEMP C/A(20,20),X(20,20),V(20,20)
     13                COMPLEX R DATA,RD,OVR SIG,
     14               $           R SUM,PARTS,R DSPLA(20)
     15                COMPLEX*16 A,X,V
     16                DIMENSION PART(2),OS PART(2,20,20)
     17                EQUIVALENCE (PARTS,PART),(OVR SIG,OS PART)
     18                DATA RTD/57.29578/
     19                DATA PI/3.141593/
     20                DATA NDIM/20/
     21        C
     22        C
     23                RD(1) = R DATA(1)
     24                R DSPLA(1) = R DATA(1)
     25                PREVP = 0.0
     26                K=2
     27                DO 28 LL=2,NR F
     28           21 K=K+1
     29                R SUM = 0.0
     30                DO 22 L=1,NR F
     31           22 R SUM = R SUM+DIDR(K,L)*R DATA(L)
     32                PARTS = CLOG(R SUM)
     33                PHASE = PART(2)
     34           24 IF (PHASE-PREVP .LT. PI) GO TO 25
     35                PHASE = PHASE-2.0*PI
     36                GO TO 24
     37           25 IF (PHASE-PREVP .GT. -PI) GO TO 26
     38                PHASE = PHASE+2.0*PI
     39                GO TO 25
     40           26 PREVP = PHASE {IF(K .EQ. 3) DPHASE = PREVP*RTD}
     41                IF(K .NE. INDEXF(LL)) GO TO 21
     42                PART(2) = PHASE
     43                RD(LL) = PARTS
     44                PART(1) = EXP(PART(1))
     45                PART(2) = PART(2)*RTD
     46           28 R DSPLA(LL) = PARTS
     47        C
     48                A(1,1) = 1.0
     49                DO 51 L=2,NR F
     50           51 A(L,1) = 0.0
     51                DO 53 LL=2,NR F
```

```
52              R SUM = 0.0
53              K = INDEXF(LL)
54              DO 52 L=1,NR F
55           52 R SUM = R SUM+DIDR(K,L)*R DATA(L)
56              DO 53 L=1,NR F
57           53 A(L,LL) = DIDR(K,L)/R SUM
58         C
59              DO 55 L=1,NR F
60              DO 55 LL=1,NR F
61           55 V(L,LL) = 0.0
62              DO 56 L=1,NR F
63           56 V(L,L) = 1.0/SIGMA(L)
64         C
65              CALL CLINEQ (A,V,X,NR F,NDIM,0,ERR)
66              PRINT 600,ERR
67          600 FORMAT ('0','CLINEQ ERR = ',F7.3,//)
68         C
69              DO 58 L=2,NR F
70              CALL CLINEQ(A,V(1,L),X(1,L),NR F,NDIM,1,ERR)
71           58 PRINT 600,ERR
72         C
73              DO 61 L=1,NR F
74              DO 61 LL=1,NR F
75           61 OVR SIG(L,LL) = X(LL,L)
76         C
77              DO 63 L=1,NR F
78              PRINT 603,(OS PART(1,L,LL),LL=1,NR F)
79              PRINT 603,(OS PART(2,L,LL),LL=1,NR F)
80          603 FORMAT (' ',10(1PE12.2))
81           63 PRINT 604
82          604 FORMAT ('0')
83         C
84              PRINT 700
85          700 FORMAT ('1','DATA VALUES--')
86              PRINT 701
87          701 FORMAT ('0',3X,'FREQ',6X,'RE(R)',3X,'IM(R)',2X,'MAG(DRDF)',1X,'A(D
88             $RDF)',9X,'RE(G)',4X,'IM(G)')
89              DO 71 L=1,NR F
90           71 PRINT 703,F(L),R DATA(L),R DSPLA(L),RD(L)
91          703 FORMAT (' ',F7.2,3X,2F8.3,3X,2F8.3,F14.5,F9.5)
92              RETURN
93         C
94              END

PRT,S INVERT.RDRDA
```

```
MORFITT*INVERT(1).RDRDA
      1              SUBROUTINE R DRDA
      2        C
      3              COMMON/DELF C/DELF
      4              COMMON/S COM/S
      5              COMMON/IPE COM/IPRNTE
      6              COMMON/ROTAT C/ROTATE
      7              COMMON/DPHAS C/DPHASE
      8              COMMON/NS COM/NR STPS(25,20)
      9              COMMON/HTS COM/HTS(51)
     10              COMMON/NRA COM/NR A
     11              COMMON/F COM/F(20)
     12              COMMON/SIGMA C/SIGMA(20)
     13              COMMON/NRF COM/NR F
     14              COMMON/DIDR C/DIDR(205,20)
     15              COMMON/IF COM/INDEXF(20)
     16              COMMON/RD COM/RD(20)
     17              COMMON/OVSIG C/OVR SIG(20,20)
     18              COMMON/FREQ NR/LFREQ
     19              COMMON/FRQ COM/FREQ
     20              COMMON/INTEGR/R11,R21,R12,R22,IR OMIT(24)
     21              COMMON/TEMP C/DR LIST(4,51),ER LIST(4,25)
     22              COMMON/OVRFLO/IOVFLO
     23              COMMON/RNORM C/R NORM(20)
     24              COMMON/DRDA C/DRDA(20,51)
     25              COMMON/DSPLA C/R DSPLA(3,20)
     26              COMMON/UR COM/UR(20),N STEPS(20)
     27              DIMENSION PART(2),U LEVEL(25)
     28              COMPLEX
     29             $           RD,
     30             $           OVR SIG,
     31             $           R11,R21,R12,R22,
     32             $           DR LIST,ER LIST,
     33             $           R NORM,DRDA,
     34             $           R DSPLA,
     35             $           CPLX I,PARTS,
     36             $           RNUM,RDEN,R(20),
     37             $           URNUM,URDEN,UR CPLX,
     38             $           DPNUM,DRDEN,DRIV,
     39             $           R SUM,R NEW(20),DRDA SV(20)
     40              EQUIVALENCE (PARTS,PART)
     41              DATA UFRCTN/0.3/
     42              DATA CPLX I/(0.0,1.0)/
     43              DATA PI/3.141593/
     44              DATA RTD/57.29578/
     45        C
     46        C
     47              NR JHF = (NR A-1)/2
     48        C
     49              DO 12 J=1,NR A
     50              DO 12 LL=1,NR F
     51           12 DRDA(LL,J) = 0.0
     52        C
```

```
52              DO 49 L=1,NR F
53              FREQ = F(L)
54           21 PRINT 201,FREQ
55          201 FORMAT (' ',F7.2)
56              LFREQ = L
57              CALL INTEG
58              IF (IOVFLO .NE. 0) RETURN
59        C
60              RNUM = -R22+CPLX I*R12
61              RDEN = -R22-CPLX I*R12
62              R(L) = RNUM/RDEN
63              R DSPLA(1,L) = R(L)
64        C
65              SUM = 0.0
66              DO 31 J=1,NR JHF
67              URNUM = -ER LIST(4,J)+CPLX I*ER LIST(3,J)
68              URDEN = -ER LIST(4,J)-CPLX I*ER LIST(3,J)
69              UR CPLX = (URNUM-(RNUM/RDEN)*URDEN)/RDEN
70              PARTS = UR CPLX
71              U LEVEL(J) = PART(1)**2+PART(2)**2
72           31 SUM = SUM+U LEVEL(J)
73              USQ = SUM
74              UR(L) = SQRT(SUM)
75        C
76              USQ MAX = (UFRCTN*SIGMA(L))**2
77              IF (USQ .LT. USQ MAX) GO TO 40
78        C
79              U MAX = 1.0E9
80              U NEXT = 0.0
81           34 SUM = 0.0
82              DO 35 J=1,NR JHF
83              U TEMP = U LEVEL(J)
84              IF (U TEMP*1.01 .LT. U MAX .AND. U TEMP .GT. U NEXT)
85             $           U NEXT = U TEMP
86              IF (U TEMP .GT. U MAX) U TEMP = U MAX
87           35 SUM = SUM+U TEMP
88              IF (SUM .LT. USQ MAX) GO TO 36
89              U MAX = U NEXT
90              U NEXT = 0.0
91              GO TO 34
92        C
93           36 DO 37 J=1,NR JHF
94              IF (U LEVEL(J)*0.99 .GT. U MAX) NR STPS(J,L) = 2*NR STPS(J,L)
95           37 CONTINUE
96        C
97              GO TO 21
98        C
99           40 N STEPS(L) = 0
100             DO 41 J=1,NR JHF
101          41 N STEPS(L) = N STEPS(L)+NR STPS(J,L)
102             DO 42 J=1,NR A
103             DRNUM = -DR LIST(4,J)+CPLX I*DR LIST(3,J)
```

```
104             DRDEN = -DR LIST(4,J)-CPLX I*DR LIST(3,J)
105             DRIV = (DRNUM-(RNUM/RDEN)*DRDEN)/RDEN
106             IF (L .EQ. 1) DRDA(1,J) = DRIV
107             DO 42 LL=2,NR F
108             K = INDEXF(LL)
109          42 DRDA(LL,J) = DRDA(LL,J)+DIDR(K,L)*DRIV
110          49 CONTINUE
111       C
112             R DSPLA(2,1) = R(1)
113             R DSPLA(3,1) = R(1)
114             R NEW(1) = R(1)-RD(1)
115             PREVP = (DPHASE-ROTATE)/RTD
116             K=2
117             DO 69 LL=2,NR F
118          61 K = K+1
119             R SUM = 0.0
120             DO 62 L=1,NR F
121          62 R SUM = R SUM+DIDR(K,L)*R(L)
122             PARTS = CLOG(R SUM)
123             PHASE = PART(2)
124          64 IF (PHASE-PREVP .LT. PI) GO TO 65
125             PHASE = PHASE-2.0*PI
126             GO TO 64
127          65 IF (PHASE-PREVP .GT. -PI) GO TO 66
128             PHASE = PHASE+2.0*PI
129             GO TO 65
130          66 PREVP = PHASE
131             IF(K .EQ. 3) ROTATE = (DPHASE-PREVP)*RTD
132             IF(K .NE. INDEXF(LL)) GO TO 61
133       C
134             PART(2) = PHASE
135             R NEW(LL) = PARTS-RD(LL)
136             DO 68 J=1,NR A
137          68 DRDA(LL,J) = DRDA(LL,J)/R SUM
138             R DSPLA(3,LL) = PARTS
139             PART(1) = EXP(PART(1))/(2.0*DELF)
140             PART(2) = PART(2)*RTD
141             R DSPLA(2,LL) = PARTS
142          69 CONTINUE
143       C
144             DO 72 L=1,NR F
145             R SUM = 0.0
146             DO 71 LL=1,NR F
147          71 R SUM = R SUM+OVR SIG(L,LL)*R NEW(LL)
148          72 R NORM(L) = R SUM
149       C
150             DO 75 J=1,NR A
151             DO 73 LL=1,NR F
152          73 DRDA SV(LL) = DRDA(LL,J)
153             DO 75 L=1,NR F
154             R SUM = 0.0
155             DO 74 LL=1,NR F
```

```
156          74 R SUM = R SUM+OVR SIG(L,LL)*DRDA SV(LL)
157          75 DRDA(L,J) = R SUM
158       C
159             S=0.0
160             DO 81 L =1,NR F
161          81 S = S+CABS(R NORM(L))**2
162             RETURN
163       C
164             END

$PRT,S INVERT.PRNTRS
```

```
MORFITT*INVERT(1).PRNTRS
      1         SUBROUTINE PRNT RS
      2   C
      3         COMMON/DSPLA C/R DSPLA(6,20)
      4         COMMON/F COM/F(20)
      5         COMMON/UR COM/UR(20),N STEPS(20)
      6         COMMON/NRF COM/NR F
      7   C
      8   C
      9         PRINT 100
     10     100 FORMAT ('0',3X,'FREQ',6X,'RE(R)',3X,'IM(R)',2X,'MAG(DRDF)',1X,'A(D
     11        $RDF)',9X,'RE(G)',4X,'IM(G)',12X,'RK ERR',3X,'RK STEPS')
     12         DO 11 L=1,NR F
     13      11 PRINT 101,F(L),(R DSPLA(K,L),K=1,6),UR(L),N STEPS(L)
     14     101 FORMAT (' ',F7.2,3X,2F8.3,3X,2F8.3,F14.5,F9.5,10X,F8.3,7X,I4)
     15         RETURN
     16   C
     17         END

@PRT,S INVERT.FWINPT
```

```
MORFITT*INVERT(1).FWINPT
      1         SUBROUTINE FW INPT
      2   C
      3         COMMON/PLOT C/IPROFP,IRPLOT
      4         COMMON/THETA C/THETA
      5         COMMON/FLD COM/AZIM,CODIP,MAGFLD
      6         COMMON/IPRNT C/IPRINT
      7         REAL MAGFLD
      8   C
      9         NAMELIST /FULLWV/THETA,AZIM,CODIP,MAGFLD,IPRINT,
     10        * IPROFP,IRPLOT
     11         DATA WAVE/'WAVE'/
     12         DATA ADAT/' &DA'/,DAT2/'TUM '/,IEND/' &EN'/,END2/'D   '/
     13         DATA FUL1/'FULL'/,FUL2/' WAV'/,FUL3/'E IN'/,FUL4/' PUT'/
     14   C
     15         IPROFP=1
     16         IRPLOT=1
     17         THETA = 22.0
     18         AZIM = 90.0
     19         CODIP = 12.0
     20         MAGFLD = 5.3E-5
     21         IPRINT = 0
     22   C
     23         READ 12, CBD
     24      12 FORMAT(1A4)
     25         IF( CBD    .NE. WAVE) PRINT 14
     26      14 FORMAT('0',' FULLWV NAMELIST IS ABSENT.--PROGRAM STOPS--.')
     27         IF( CBD    .NE. WAVE) GO TO 999
     28         PRINT 10,FUL1,FUL2,FUL3,FUL4
     29      10 FORMAT(' ',4A4/)
     30         IF( CBD .EQ. WAVE) PRINT 13,WAVE
     31      13 FORMAT(' ',A4)
     32         READ(5,FULLWV)
     33         PRINT 16,ADAT,DAT2
     34      16 FORMAT(' ',2A4)
     35         PRINT 11,THETA,AZIM,CODIP,MAGFLD,IPRINT,IPROFP,IRPLOT
     36      11 FORMAT(' ',' THETA = ',F8.3,' DEGREES',' AZIM = ',F8.3,
     37        * ' DEGREES',' CODIP = ',F8.3,' DEGREES',
     38        * ' MAGFLD = ',1PE10.3,/ ' IPRINT = ',I2,' IPROFP = ',I2,
     39        * ' IRPLOT = ',I2)
     40         PRINT 17,IEND,END2
     41      17 FORMAT(' ',2A4,/)
     42   C
     43         CALL INIT T
     44         RETURN
     45   C
     46    999 STOP
     47   C
     48         END

@PRT,S INVERT.INTEG
```

```
MORFITT*INVERT(1).INTEG
      1          SUBROUTINE INTEG
      2    C
      3          COMMON/NS COM/NP STPS(25,20)
      4          COMMON/DLTAH C/DELTAH
      5          COMMON/HTS COM/HTS(51)
      6          COMMON/NRA COM/NR A
      7          COMMON/IPRNT C/IPRINT
      8          COMMON/FREQ NR/LFREQ
      9          COMMON/JAY COM/JAY
     10          COMMON/TEMP C/DR LIST(8,51),ER LIST(8,25),
     11         $           A STOR(32,49),
     12         $           R0(56),HDELR0(56),DELR1(56),DELR2(56),
     13         $           X0(8),HDELX0(8),DELX1(8),DELX2(8),
     14         $           R SAVE(56),
     15         $           ERROR(8),
     16         $           A TEMP(32),A PROD(32),
     17         $           TEMPV(8)
     18          COMMON/INTEGR/R(56)
     19          COMMON/DRDH C/DRDH(56)
     20          COMMON/X COM/X(8)
     21          COMMON/DXDH C/DXDH(8)
     22          COMMON/EN COLL/HT
     23          COMMON/WN COM/WAVE NR
     24          COMMON/OVRFLO/IOVFLO
     25          COMMON/IEW COM/IEW
     26          DATA DHMIN/0.01/
     27          DATA RTOL/0.03/
     28    C
     29    C
     30          IOVFLO = 0
     31          CALL INIT S
     32          JAY = 1
     33          HT = HTS(1)
     34          CALL COEFFS
     35          CALL INITLI
     36          CALL SET P DR
     37    C
     38          NR EQ = 56-20*IEW
     39          N6N8 = 8-2*IEW
     40          N25N41 = 41-16*IEW
     41          N18N32 = 32-14*IEW
     42    C
     43          IF (IPRINT .NE. 0) PRINT 900
     44          IF (IPRINT .NE. 0) PRINT 902,HT,(R(I),I=1,N6N8)
     45    C
     46          NRAM1 = NR A-1
     47          NR JHF = (NR A-1)/2
     48          DO 69 J=2,NRAM1,2
     49          JM1 = J-1
     50          JHF = J/2
     51          JAY = JM1
```

```
      CALL XFER (R,R SAVE,NR EQ)
      CALL XFER (R,X,N6N8)
      HT = HTS(JM1)
   31 NS = NR STPS(JHF,LFREQ)
      DELH = -2.0*DELTAH/FLOAT(NS)
      DH = DELH*WAVE NR/2.0
      HDH = 0.5*DH
      TDH = 2.0*DH
C
      NSOVR2 = NS/2
      NSOVR4 = NS/4
      DO 49 K=1,NSOVR2
      CALL R DERIV
      CALL X DERIV
      IF (IOVFLO .NE. 0) GO TO 60
C
      DO 32 I=1,NR EQ
      R0(I) = R(I)
      HDELR0(I) = DRDH(I)*HDH
   32 R(I) = R0(I)+HDELR0(I)
C
      DO 33 I=1,N6N8
      X0(I) = X(I)
      HDELX0(I) = DXDH(I)*DH
   33 X(I) = X0(I)+HDELX0(I)
C
      HT = HT+0.5*DELH
      CALL COEFFS
      CALL R DERIV
      IF (IOVFLO .NE. 0) GO TO 60
C
      DO 34 I=1,NR EQ
      DELR1(I) = DRDH(I)*DH
   34 R(I) = R0(I)+0.5*DELR1(I)
C
      CALL R DERIV
      IF (IOVFLO .NE. 0) GO TO 60
C
      DO 35 I=1,NR EQ
      DELR2(I) = DRDH(I)*DH
   35 R(I) = R0(I)+DELR2(I)
C
      HT = HT+0.5*DELH
      CALL COEFFS
      CALL R DERIV
      CALL X DERIV
      IF (IOVFLO .NE. 0) GO TO 60
C
      DO 36 I=1,NR EQ
      HDELR3 = DRDH(I)*HDH
      DELR4 = (HDELR0(I)+DELR1(I)+DELR2(I)+HDELR3)/3.0
   36 R(I) = R0(I)+DELR4
```

```
104   C
105         DO 37 I=1,N6N8
106         DELX1(I) = DXDH(I)*TDH
107      37 X(I) = XO(I)+0.5*DELX1(I)
108   C
109         IF (NS .NE. 2) GO TO 40
110         IF (JM1 .GT. 1) CALL XFER (R(N6N8+1),A STOR(1,JM1-1),N13N32)
111         CALL XFER (R(N25N41),DR LIST(1,JM1),N6N8)
112         CALL SET P DR
113         JAY = J
114   C
115      40 CALL R DERIV
116         CALL X DERIV
117         IF (IOVFLO .NE. 0) GO TO 60
118   C
119         DO 42 I=1,NR EQ
120         RO(I) = R(I)
121         HDELRO(I) = DRDH(I)*HDH
122      42 R(I) = RO(I)+HDELRO(I)
123   C
124         DO 43 I=1,N6N8
125         DELX2(I) = DXDH(I)*TDH
126      43 X(I) = XO(I)+DELX2(I)
127   C
128         HT = HT+0.5*DELH
129         CALL COEFFS
130         CALL R DERIV
131         IF (IOVFLO .NE. 0) GO TO 60
132   C
133         DO 44 I=1,NR EQ
134         DELR1(I) = DRDH(I)*DH
135      44 R(I) = RO(I)+0.5*DELR1(I)
136   C
137         CALL R DERIV
138         IF (IOVFLO .NE. 0) GO TO 60
139   C
140         DO 45 I=1,NR EQ
141         DELR2(I) = DRDH(I)*DH
142      45 R(I) = RO(I)+DELR2(I)
143   C
144         HT = HT+0.5*DELH
145         CALL COEFFS
146         CALL R DERIV
147         CALL X DERIV
148         IF (IOVFLO .NE. 0) GO TO 60
149   C
150         DO 46 I=1,NR EQ
151         HDELR3 = DRDH(I)*HDH
152         DELR4 = (HDELRO(I)+DELR1(I)+DELR2(I)+HDELR3)/3.0
153      46 R(I) = RO(I)+DELR4
154   C
155         DO 47 I=1,N6N8
```

```
156         HDELX3 = DXDH(I)*DH
157         DELX4 = (HDELX0(I)+DELX1(I)+DELX2(I)+HDELX3)/3.0
158      47 X(I) = X0(I)+DELX4
159   C
160         IF (K .NE. NSOVR4) GO TO 49
161         IF (JM1 .GT. 1) CALL XFER (R(N6N8+1),A STOR(1,JM1-1),N18N32)
162         CALL XFER (R(N25N41),DR LIST(1,JM1),N6N8)
163         CALL SET P DR
164         JAY = J
165      49 CONTINUE
166   C
167         IF (IPRINT .NE. 0) PRINT 900
168         IF (IPRINT .NE. 0) PRINT 901,(X(I),I=1,N6N8)
169         IF (IPRINT .NE. 0) PRINT 902,HT,(R(I),I=1,N6N8)
170   C
171         DO 51 I=1,N6N8
172         ERROR(I) = R(I)-X(I)
173         IF (ABS(ERROR(I)) .GT. RTOL) GO TO 61
174      51 CONTINUE
175         GO TO 65
176   C
177      60 IF (IPRINT .NE. 0) PRINT 900
178         IF (IPRINT .NE. 0) PRINT 902,HT,(R(I),I=1,N6N8)
179      61 IF (IPRINT .NE. 0) PRINT 600
180     600 FORMAT (' ','OVRFLO')
181         IF (ABS(DELH/2.0) .LT. DHMIN) RETURN
182         IOVFLO = 0
183         CALL XFER (R SAVE,R,NR EQ)
184         CALL XFER (R,X,N6N8)
185         NR STPS(JHF,LFREQ) = 2*NR STPS(JHF,LFREQ)
186         HT = HTS(JM1)
187         JAY = JM1
188         CALL COEFFS
189         GO TO 31
190   C
191      65 CALL XFER (ERROR,ER LIST(1,JHF),N6N8)
192         IF (J .GT. 1) CALL XFER (R(N6N8+1),A STOR(1,J-1),N18N32)
193         CALL XFER (R(N25N41),DR LIST(1,J),N6N8)
194         CALL SET P DR
195      69 CONTINUE
196   C
197         J = NRAM1
198         CALL XFER (R(N25N41),DR LIST(1,J+1),N6N8)
199   C
200         J = J-1
201         CALL XFER(A STOR(1,J),A PROD,N18N32)
202         JHF = NR JHF-1
203   C
204      72 CALL MULTVC(A PROD,DR LIST(1,J),TEMPV)
205         CALL XFER(TEMPV,DR LIST(1,J),N6N8)
206         IF (J .NE. JHF*2) GO TO 73
207         CALL MULTVC (A PROD,ER LIST(1,JHF),TEMPV)
```

```
      CALL XFER (TEMPV,ER LIST(1,JHF),N6N8)
      JHF = JHF-1
   73 J = J-1
      IF (J .LE. 0) GO TO 74
      CALL MULT(A PROD,A STOR(1,J),A TEMP)
      CALL XFER(A TEMP,A PROD,N18N32)
      GO TO 72
   74 CONTINUE
      IF (IEW .EQ. 0) RETURN
C
      IF (IEW .NE. 1) GO TO 85
      DO 82 II=1,4
      I = 9-II
      R(I) = R(I-2)
C
      DO 81 J=1,NR A
   81 DR LIST(I,J) = DR LIST(I-2,J)
C
      DO 82 J=1,NR JHF
   82 ER LIST(I,J) = ER LIST(I-2,J)
      RETURN
C
   85 R(7) = -(R(1)+R(3)+1.0)
      R(8) = -(R(2)+R(4))
      R(5) = R(2)-R(4)
      R(6) = R(3)-R(1)
C
      DO 86 J=1,NR A
      DR LIST(7,J) = -(DR LIST(1,J)+DR LIST(3,J))
      DR LIST(8,J) = -(DR LIST(2,J)+DR LIST(4,J))
      DR LIST(5,J) = DR LIST(2,J)-DR LIST(4,J)
   86 DR LIST(6,J) = DR LIST(3,J)-DR LIST(1,J)
C
      DO 87 J=1,NR JHF
      ER LIST(7,J) = -(ER LIST(1,J)+ER LIST(3,J))
      ER LIST(8,J) = -(ER LIST(2,J)+ER LIST(4,J))
      ER LIST(5,J) = ER LIST(2,J)-ER LIST(4,J)
   87 ER LIST(6,J) = ER LIST(3,J)-ER LIST(1,J)
      RETURN
C
  900 FORMAT ('0')
  901 FORMAT (' ',10X,4(2X,2F9.5))
  902 FORMAT (' ',F10.2,4(2X,2F9.5))
C
      END

@PRT,S INVERT.ENNU
```

```
MORFITT*INVERT(1).ENNU
      1          SUBROUTINE EN NU
      2  C
      3          COMMON/DLTAH C/DELTAH
      4          COMMON/JAY COM/J
      5          COMMON/HTS COM/HTS(51)
      6          COMMON/ALPHA C/ALPHA(51)
      7          COMMON/EXPNU C/COEFNU,EXPNU
      8          COMMON/EN COLL/HT,EN,NU
      9          COMMON/W COM/WT,WB
     10          REAL NU
     11  C
     12  C
     13          WT = (HT-HTS(J+1))/DELTAH
     14          WB = (HTS(J)-HT)/DELTAH
     15          EN = EXP(WT*ALPHA(J)+WB*ALPHA(J+1))
     16          NU = COEFNU*EXP(EXPNU*HT)
     17          RETURN
     18  C
     19          END

@PRT,S INVERT.INITLI
```

```
MORFITT*INVERT(1).INITLI
      1         SUBROUTINE INITLI
      2   C
      3         COMMON/IPRNT C/IPRINT
      4         COMMON/CS COM/C,S,CSQ
      5         COMMON/M COM/M11,M22,M33,M21,M31,M23
      6         COMMON/INTEGR/R11,R12,R22,IR SKIP(24),DR11,DR12,DR22
      7         COMPLEX
      8        $           M11,M22,M33,M21,M31,M23,
      9        $           R11,R12,R22,
     10        $           DR11,DR12,DR22,
     11        $           B4,B2,B0,HALF B2,ROOT,Q(2),
     12        $           D11,D12,D13,D31,D32,D33,
     13        $           DEN,P(2),T(2),FACTOR,
     14        $           DB4,DB2,DB0,DQ(2),
     15        $           DD11,DD12,DD13,DD31,DD32,DD33,
     16        $           DDEN,DP(2),DT(2)
     17   C
     18   C
     19         B4 = 1.0+M33
     20         B2 = -(CSQ+M33)*(1.0+M11)-M31**2-(1.0+M33)*(CSQ+M22)+M23**2
     21         B0 = (1.0+M11)*(CSQ+M22)*(CSQ+M33)-2.0*M21*M31*M23
     22        $           -(1.0+M11)*M23**2+(CSQ+M22)*M31**2+(CSQ+M33)*M21**2
     23   C
     24         HALF B2 = B2/2.0
     25         ROOT = CSQRT(HALF B2**2-B4*B0)
     26         Q(1) = CSQRT((-HALF B2+ROOT)/B4)
     27         Q(2) = CSQRT((-HALF B2-ROOT)/B4)
     28   C
     29         IF (IPRINT .NE. 0) PRINT 200,Q(1),Q(2)
     30     200 FORMAT ('0','Q = ',2(2X,2E13.4),/)
     31   C
     32         DO 31 K=1,2
     33         D11 = 1.0+M11-Q(K)**2
     34         D12 = -M21
     35         D13 = -M31+Q(K)*S
     36         D31 = M31+Q(K)*S
     37         D32 = M23
     38         D33 = CSQ+M33
     39         DEN = D11*D33-D13*D31
     40         P(K) = (-D12*D33+D13*D32)/DEN
     41      31 T(K) = Q(K)*P(K)-S*(-D11*D32+D12*D31)/DEN
     42   C
     43         DB4 = M33
     44         DB2 = -M33*(1.0+M11)-(CSQ+M33)*M11-2.0*M31**2
     45        $           -M33*(CSQ+M22)-(1.0+M33)*M22+2.0*M23**2
     46         DB0 = M11*(CSQ+M22)*(CSQ+M33)+(1.0+M11)*M22*(CSQ+M33)
     47        $           +(1.0+M11)*(CSQ+M22)*M33-6.0*M21*M31*M23
     48        $           -(2.0+3.0*M11)*M23**2
     49        $           +(2.0*CSQ+3.0*M22)*M31**2
     50        $           +(2.0*CSQ+3.0*M33)*M21**2
     51   C
```

```
      DO 41 K=1,2
      DQ(K) = -((DB4*Q(K)**2+DB2)*Q(K)**2+DB0)
     $           /((4.0*B4*Q(K)**2+2.0*B2)*Q(K))
      DD11 = M11-2.0*Q(K)*DQ(K)
      DD12 = -M21
      DD13 = -M31+DQ(K)*S
      DD31 = M31+DQ(K)*S
      DD32 = M23
      DD33 = M33
      DDEN = DD11*D33+D11*DD33-DD13*D31-D13*DD31
      DP(K) = -DDEN*P(K)/DEN
     $           +(-DD12*D33-D12*DD33+DD13*D32+D13*DD32)/DEN
   41 DT(K) = DQ(K)*P(K)+Q(K)*DP(K)
     $           -S*(-DD11*D32-D11*DD32+DD12*D31+D12*DD31
     $           -(-D11*D32+D12*D31)*DDEN/DEN)/DEN
C
      DEN = (T(1)*C+P(1))*(C+Q(2))-(T(2)*C+P(2))*(C+Q(1))
      FACTOR = 2.0/DEN
      R11 = (C*(T(1)-T(2))+T(1)*Q(2)-T(2)*Q(1))*FACTOR
      R22 = (C*(T(1)-T(2))+P(1)-P(2))*FACTOR
      R12 = (Q(2)-Q(1))*FACTOR
C
      DDEN = (DT(1)*C+DP(1))*(C+Q(2))+(T(1)*C+P(1))*DQ(2)
     $           -(DT(2)*C+DP(2))*(C+Q(1))-(T(2)*C+P(2))*DQ(1)
      DR11 = -R11*DDEN/DEN
     $           +(C*(DT(1)-DT(2))
     $           +DT(1)*Q(2)+T(1)*DQ(2)-DT(2)*Q(1)-T(2)*DQ(1))*FACTOR
      DR22 = -R22*DDEN/DEN
     $           +(C*(DT(1)-DT(2))+DP(1)-DP(2))*FACTOR
      DR12 = -R12*DDEN/DEN
     $           +(DQ(2)-DQ(1))*FACTOR
C
      R11 = C*R11-1.0
      R22 = C*R22-1.0
      R12 = C*R12
C
      DR11 = C*DR11
      DR22 = C*DR22
      DR12 = C*DR12
      RETURN
C
      END

@PRT,S INVERT.COEFFS
```

```
MORFITT*INVERT(1).COEFFS
      1         SUBROUTINE COEFFS
      2  C
      3         COMMON/THETA C/THETA
      4         COMMON/FLD COM/AZIM,CODIP,MAGFLD
      5         COMMON/FRQ COM/FREQ
      6         COMMON/EN COLL/HT,EN,COLL
      7         COMMON/WN COM/WAVE NR
      8         COMMON/CS COM/C,S,CSQ
      9         COMMON/M COM/M11,M22,M33,M21,M31,M23
     10         COMMON/SD COM/D11A,S11B,D11B,S21,D21,S22,D22
     11         COMMON/DSD COM/DD11A,DS11B,DD11B,DS21,DD21,DS22,DD22
     12         COMMON/IEW COM/IEW
     13         COMPLEX
     14        $           M11,M22,M33,M21,M31,M23,
     15        $           D11A,S11B,D11B,S21,D21,S22,D22,
     16        $           DD11A,DS11B,DD11B,DS21,DD21,DS22,DD22,
     17        $           U,USQ,D,IUD,
     18        $           M23D,M31D,
     19        $           M2323D,M3131D,M2331D,
     20        $           T14 OVC,T32 OVC,CT41,T11,T31,T34 OVC,
     21        $           DT14OC,DT32OC,DCT41,DT11,DT31,DT34OC
     22         REAL MAGFLD,
     23        $           MY,NY,IUD PRT,
     24        $           MSQYSQ,NSQYSQ,MNYSQ
     25         DIMENSION U PARTS(2),USQ PRT(2),D PARTS(2),IUD PRT(2)
     26         EQUIVALENCE (U,U PARTS),(USQ,USQ PRT),(D,D PARTS),(IUD,IUD PRT)
     27         DATA PI/3.141593/
     28         DATA RTD/57.29578/
     29         DATA COEFF X/3.182357E3/
     30         DATA COEFF Y/1.758796E11/
     31         DATA VEL LT/2.997928E5/
     32  C
     33  C
     34         CALL EN NU
     35  C
     36         X = COEF EN*EN
     37         Z = COLL*OV OMGA
     38         U PARTS(1) = 1.0
     39         U PARTS(2) = -Z
     40         USQ PRT(1) = 1.0-Z**2
     41         USQ PRT(2) = -Z-Z
     42         D = -X/(U*(USQ-YSQ))
     43         IF(ABS(D PARTS(1)) .LT. 1.0E-30 .AND.
     44        $           ABS(D PARTS(2)) .LT. 1.0E-30) D = 0.0
     45         IUD PRT(1) = Z*D PARTS(1)-D PARTS(2)
     46         IUD PRT(2) = D PARTS(1)+Z*D PARTS(2)
     47         M11 = USQ*D
     48         M22 = M11-MSQYSQ*D
     49         M33 = M11-NSQYSQ*D
     50         M21 = NY*IUD
     51         M31 = -MY*IUD
```

```
      M23 = -MNYSQ*D
C
      D = 1.0/(1.0+M33)
      M23D = M23*D
      M31D = M31*D
      T14 OVC = OVR C-SSQ OVC*D
      M2323D = M23*M23D
      T32 OVC = (CSQ+M22-M2323D)*OVR C
      M3131D = M31*M31D
      CT41 = C*(1.0+M11+M3131D)
      T11 = -S*M31D
      M2331D = M23*M31D
      T31 = M2331D-M21
      T34 OVC = S OVR C*M23D
C
      D11A = T11+T11
      S11B = T14 OVC+CT41
      D11B = T14 OVC-CT41
      S21 = T34 OVC+T31
      D21 = T34 OVC-T31
      S22 = C+T32 OVC
      D22 = C-T32 OVC
C
      DT140C     = -SSQ OVC*D*(D-1.0)
      DT320C     = (M22-M2323D*(D+1.0))*OVR C
      DCT41     = C*(M11+M3131D*(D+1.0))
      DT11     = T11*D
      DT31     = M2331D*(D+1.0)-M21
      DT340C     = T34 OVC*D
C
      DD11A = DT11+DT11
      DS11B = DT140C+DCT41
      DD11B = DT140C-DCT41
      DS21 = DT340C+DT31
      DD21 = DT340C-DT31
      DS22 = DT320C
      DD22 = -DT320C
      RETURN
C
C
      ENTRY INIT T
      IEW = 1
      DIR CS M = SIN(CODIP/RTD)*SIN(AZIM/RTD)
      DIR CS N = -COS(CODIP/RTD)
      C = COS(THETA/RTD)
      S = SIN(THETA/RTD)
      OVR C = 1.0/C
      CSQ = C**2
      S OVR C = S/C
      SSQ OVC = S**2/C
      RETURN
C
```

```
104   C
105           ENTRY INIT S
106           OMEGA = 2.0*PI*FREQ*1000.0
107           WAVE NR = OMEGA/VEL LT
108           COEF EN = COEFF X*1.0E06/OMEGA**2
109           OV OMGA = 1.0/OMEGA
110           Y = -COEFF Y*MAGFLD/OMEGA
111           YSQ = Y**2
112           MY = DIR CS M*Y
113           NY = DIR CS N*Y
114           MSQYSQ = DIR CS M**2*YSQ
115           NSQYSQ = DIR CS N**2*YSQ
116           MNYSQ = DIR CS M*DIR CS N*YSQ
117           RETURN
118   C
119           END

@PRT,S INVERT.DIFFEQ
```

```
MORFITT*INVERT(1).DIFFEQ
      1         SUBROUTINE DIFF EQ
      2   C
      3         COMMON/SD COM/D11A,S11B,D11B,S21,D21,S22,D22
      4         COMMON/DSD COM/DD11A,DS11B,DD11B,DS21,DD21,DS22,DD22
      5         COMMON/W COM/WT,WB
      6         COMMON/INTEGR/R(36)
      7         COMMON/DRDH C/DRDH(36)
      8         COMMON/X COM/X(6)
      9         COMMON/DXDH C/DXDH(6)
     10         COMMON/ORDER C/R11,R12,R22,P(3,3),DR(6),
     11        $           DER(3),PDER(3,3),DDER(6)
     12         COMMON/OVRFLO/IOVFLO
     13         COMPLEX
     14        $           D11A,S11B,D11B,S21,D21,S22,D22,
     15        $           DD11A,DS11B,DD11B,DS21,DD21,DS22,DD22,
     16        $           R11,R12,R22,P,DR,
     17        $           DER,PDER,DDER,
     18        $           B11,B12,B22,R12R12,C12,
     19        $           DB11,DB22,DB12,DC12,
     20        $           X11,X12,X21,X22,
     21        $           G11,G12,G21,G22,G23,G32,G33,
     22        $           DTEMP(3)
     23         DIMENSION R MTRX(36),DERIV(36)
     24         EQUIVALENCE (R11,R MTRX),(DER,DERIV)
     25   C
     26   C
     27         ENTRY R DERIV
     28         CALL XFER (R,R MTRX,36)
     29         IXFLAG = 0
     30   C
     31      20 DO 21 I=1,6
     32         IF (ABS(R MTRX(I)) .GT. 9.0) GO TO 90
     33      21 CONTINUE
     34   C
     35         B11 = R11*(D11A-D11B)
     36         B12 = R12*D21
     37         B22 = R22*D22
     38         R12R12 = R12*R12
     39         C12 = R12*S21
     40   C
     41         DER(1)= (B11+B12+B12-S11A-S11B)*R11+R12R12*D22+C12+C12-D11A-D11B
     42         DER(2)= (B11+B12+B22      -S11B-S22)*R12+(R11+D21+S21)*(R22+1.0)
     43         DER(3)= (B12+B12+B22-S22-S22)*R22+R12R12*(D11A-D11B)+B12+B12+D22
     44         IF (IXFLAG .NE. 0) GO TO 60
     45   C
     46         X11 = -S11B+B11+B12
     47         X12 = S21+R11*D21+R12*D22
     48         X21 = D21+R12*(D11A-D11B)+R22*D21
     49         X22 = -S22+B12+B22
     50   C
     51         G11 = X11+X11
```

```
      G12 = X12+X12
      G21 = X21
      G22 = X11+X22
      G23 = X12
      G32 = X21+X21
      G33 = X22+X22
C
      DO 31 L=1,3
      PDER(1,L) = G11*P(1,L)+G12*P(2,L)
      PDER(2,L) = G21*P(1,L)+G22*P(2,L)+G23*P(3,L)
   31 PDEP(3,L) =             G32*P(2,L)+G33*P(3,L)
C
      DB11 = R11*(DD11A-DD11B)
      DB22 = R22*DD22
      DB12 = R12*DD21
      DC12 = R12*DS21
C
      DTEMP(1) = (DB11+DB12+DB12-DS11B-DS11B)*R11
     $           +R12R12*DD22+DC12+DC12-DD11A-DD11B
      DTEMP(2) = (DB11+DB12+DB22-DS11B-DS22)*R12
     $           +(R11*DD21+DS21)*(R22+1.0)
      DTEMP(3) = (DB12+DB12+DB22-DS22-DS22)*R22
     $           +R12R12*(DD11A-DD11B)+DB12+DB12+DD22
C
      DDER(1)= DTEMP(1)*WT   +G11*DR(   1)+G12*DR(   2)
      DDER(4)= DTEMP(1)*WB   +G11*DR(   4)+G12*DR(   5)
      DDER(2)= DTEMP(2)*WT   +G21*DR(   1)+G22*DR(   2)+G23*DR(   3)
      DDER(5)= DTEMP(2)*WB   +G21*DR(   4)+G22*DR(   5)+G23*DR(   6)
      DDER(3)= DTEMP(3)*WT                 +G32*DR(   2)+G33*DR(   3)
      DDER(6)= DTEMP(3)*WB                 +G32*DR(   5)+G33*DR(   6)
C
C                                (A+I*B)/I = B-I*A
      DO 41 I=1,35,2
      DRDH(I) = DERIV(I+1)
   41 DRDH(I+1) = -DERIV(I)
      RETURN
C
C
      ENTRY X DERIV
      CALL XFER (X,R MTRX,6)
      IXFLAG = 1
      GO TO 20
C
   60 DO 61 I=1,5,2
      DXDH(I) = DERIV(I+1)
   61 DXDH(I+1) = -DERIV(I)
      RETURN
C
C
      ENTRY SET P DR
      CALL XFER (R,R MTRX,36)
      DO 72 L=1,3
```

```
104           DO 71 I=1,3
105        71 P(I,L) = 0.0
106        72 P(L,L) = 1.0
107     C
108           DO 74 I=1,3
109           DR(I) = DR(I+3)
110        74 DR(I+3) = 0.0
111           CALL XFER (R MTRX,R,36)
112           WT = 1.0
113           WB = 0.0
114           RETURN
115     C
116        90 IOVFLO = 1
117           RETURN
118     C
119           END
120
121
122
```

```
&PRT,S INVERT.MTXFCT
```

```
MORFITT*INVERT(1).MTXFCT
      1         SUBROUTINE MTX FCT
      2   C
      3         DIMENSION A(1),B(1)
      4         COMPLEX C(3,3),D(3,3),E(3,3),
      5        $           F(3,3),G(3),H(3),SUM
      6   C
      7   C
      8         ENTRY XFER (A,B,N)
      9         DO 11 J=1,N
     10      11 B(J) = A(J)
     11         RETURN
     12   C
     13   C
     14         ENTRY MULT (C,D,E)
     15         DO 22 J=1,3
     16         DO 22 JJ=1,3
     17         SUM = 0.0
     18         DO 21 JJJ=1,3
     19      21 SUM = SUM+C(J,JJJ)*D(JJJ,JJ)
     20      22 E(J,JJ) = SUM
     21         RETURN
     22   C
     23   C
     24         ENTRY MULTVC (F,G,H)
     25         DO 32 J=1,3
     26         SUM = 0.0
     27         DO 31 JJ=1,3
     28      31 SUM = SUM+F(J,JJ)*G(JJ)
     29      32 H(J) = SUM
     30         RETURN
     31   C
     32         END

@PRT,S INVERT.DGELB
```

```
MORFITT*INVERT(1).DGELB
      1          SUBROUTINE DGELB(R,A,M,N,MUD,MLD,EPS,IER)                        DELB
      2    C                                                                      DELB
      3    C     ..................................................................DELB
      4    C                                                                      DELB
      5    C        SUBROUTINE DGELB                                              DELB
      6    C                                                                      DELB
      7    C        PURPOSE                                                       DELB
      8    C           TO SOLVE A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS WITH A  DELB
      9    C           COEFFICIENT MATRIX OF BAND STRUCTURE.                      DELB
     10    C                                                                      DELB
     11    C        USAGE                                                         DELB
     12    C           CALL DGELB(R,A,M,N,MUD,MLD,EPS,IER)                        DELB
     13    C                                                                      DELB
     14    C        DESCRIPTION OF PARAMETERS                                     DELB
     15    C           R       - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX   DELB
     16    C                     (DESTROYED). ON RETURN R CONTAINS THE SOLUTION   DELB
     17    C                     OF THE EQUATIONS.                                DELB
     18    C           A       - DOUBLE PRECISION M BY M COEFFICIENT MATRIX WITH  DELB
     19    C                     BAND STRUCTURE (DESTROYED).                      DELB
     20    C           M       - THE NUMBER OF EQUATIONS IN THE SYSTEM.           DELB
     21    C           N       - THE NUMBER OF RIGHT HAND SIDE VECTORS.           DELB
     22    C           MUD     - THE NUMBER OF UPPER CODIAGONALS (THAT MEANS      DELB
     23    C                     CODIAGONALS ABOVE MAIN DIAGONAL).                DELB
     24    C           MLD     - THE NUMBER OF LOWER CODIAGONALS (THAT MEANS      DELB
     25    C                     CODIAGONALS BELOW MAIN DIAGONAL).                DELB
     26    C           EPS     - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS DELB
     27    C                     RELATIVE TOLERANCE FOR TEST ON LOSS OF           DELB
     28    C                     SIGNIFICANCE.                                    DELB
     29    C           IER     - RESULTING ERROR PARAMETER CODED AS FOLLOWS       DELB
     30    C                     IER=0  - NO ERROR,                               DELB
     31    C                     IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAME-DELB
     32    C                              TERS M,MUD,MLD OR BECAUSE OF PIVOT ELEMENTDELB
     33    C                              AT ANY ELIMINATION STEP EQUAL TO 0,     DELB
     34    C                     IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-DELB
     35    C                              CANCE INDICATED AT ELIMINATION STEP K+1,DELB
     36    C                              WHERE PIVOT ELEMENT WAS LESS THAN OR    DELB
     37    C                              EQUAL TO THE INTERNAL TOLERANCE EPS TIMES DELB
     38    C                              ABSOLUTELY GREATEST ELEMENT OF MATRIX A.DELB
     39    C                                                                      DELB
     40    C        REMARKS                                                       DELB
     41    C           BAND MATRIX A IS ASSUMED TO BE STORED ROWWISE IN THE FIRST DELB
     42    C           ME SUCCESSIVE STORAGE LOCATIONS OF TOTALLY NEEDED MA       DELB
     43    C           STORAGE LOCATIONS, WHERE                                   DELB
     44    C             MA=M*MC-ML*(ML+1)/2    AND    ME=MA-MU*(MU+1)/2   WITH DELB
     45    C             MC=MIN(M,1+MUD+MLD),  ML=MC-1-MLD,  MU=MC-1-MUD.       DELB
     46    C           RIGHT HAND SIDE MATRIX R IS ASSUMED TO BE STORED COLUMNWISE DELB
     47    C           IN N*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN SOLUTION     DELB
     48    C           MATRIX R IS STORED COLUMNWISE TOO.                         DELB
     49    C           INPUT PARAMETERS M, MUD, MLD SHOULD SATISFY THE FOLLOWING  DELB
     50    C           RESTRICTIONS      MUD NOT LESS THAN ZERO                   DELB
     51    C                             MLD NOT LESS THAN ZERO                   DELB
```

```
C                              MUD+MLD NOT GREATER THAN 2*M-2.                  DELB
C          NO ACTION BESIDES ERROR MESSAGE IER=-1 TAKES PLACE IF THESE          DELB
C          RESTRICTIONS ARE NOT SATISFIED.                                      DELB
C          THE PROCEDURE GIVES RESULTS IF THE RESTRICTIONS ON INPUT             DELB
C          PARAMETERS ARE SATISFIED AND IF PIVOT ELEMENTS AT ALL                DELB
C          ELIMINATION STEPS ARE DIFFERENT FROM 0. HOWEVER WARNING              DELB
C          IER=K - IF GIVEN - INDICATES POSSIBLE LOSS OF SIGNIFICANCE.          DELB
C          IN CASE OF A WELL SCALED MATRIX A AND APPROPRIATE TOLERANCE          DELB
C          EPS, IER=K MAY BE INTERPRETED THAT MATRIX A HAS THE RANK K.          DELB
C          NO WARNING IS GIVEN IF MATRIX A HAS NO LOWER CODIAGONAL.             DELB
C                                                                               DELB
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                           DELB
C          NONE                                                                 DELB
C                                                                               DELB
C       METHOD                                                                  DELB
C          SOLUTION IS DONE BY MEANS OF GAUSS ELIMINATION WITH                  DELB
C          COLUMN PIVOTING ONLY, IN ORDER TO PRESERVE BAND STRUCTURE            DELB
C          IN REMAINING COEFFICIENT MATRICES.                                   DELB
C                                                                               DELB
C     ..........................................................................DELB
C                                                                               DELB
C                                                                               DELB
C                                                                               DELB
      DIMENSION R(1),A(1)                                                       DELB
      DOUBLE PRECISION R,A,PIV,TB,TOL                                           DELB
C                                                                               DELB
C     TEST ON WRONG INPUT PARAMETERS                                            DELB
      IF(MLD)47,1,1                                                             DELB
    1 IF(MUD)47,2,2                                                             DELB
    2 MC=1+MLD+MUD                                                              DELB
      IF(MC+1-M-M)3,3,47                                                        DELB
C                                                                               DELB
C     PREPARE INTEGER PARAMETERS                                                DELB
C        MC=NUMBER OF COLUMNS IN MATRIX A                                       DELB
C        MU=NUMBER OF ZEROS TO BE INSERTED IN FIRST ROW OF MATRIX A             DELB
C        ML=NUMBER OF MISSING ELEMENTS IN LAST ROW OF MATRIX A                  DELB
C        MR=INDEX OF LAST ROW IN MATRIX A WITH MC ELEMENTS                      DELB
C        MZ=TOTAL NUMBER OF ZEROS TO BE INSERTED IN MATRIX A                    DELB
C        MA=TOTAL NUMBER OF STORAGE LOCATIONS NECESSARY FOR MATRIX A            DELB
C        NM=NUMBER OF ELEMENTS IN MATRIX R                                      DELB
    3 IF(MC-M)5,5,4                                                             DELB
    4 MC=M                                                                      DELB
    5 MU=MC-MUD-1                                                               DELB
      ML=MC-MLD-1                                                               DELB
      MR=M-ML                                                                   DELB
      MZ=(MU*(MU+1))/2                                                          DELB
      MA=M*MC-(ML*(ML+1))/2                                                     DELB
      NM=N*M                                                                    DELB
C                                                                               DELB
C     MOVE ELEMENTS BACKWARD AND SEARCH FOR ABSOLUTELY GREATEST ELEMENT         DELB
C     (NOT NECESSARY IN CASE OF A MATRIX WITHOUT LOWER CODIAGONALS)             DELB
      IFR=0                                                                     DELB
```

```
      PIV=0.D0                                                          DELB
      IF(MLD)14,14,6                                                    DELB
    6 JJ=MA                                                             DELB
      J=MA-MZ                                                           DELB
      KST=J                                                             DELB
      DO 9 K=1,KST                                                      DELB
      TB=A(J)                                                           DELB
      A(JJ)=TB                                                          DELB
      TB=DABS(TB)                                                       DELB
      IF(TB-PIV)8,8,7                                                   DELB
    7 PIV=TB                                                            DELB
    8 J=J-1                                                             DELB
    9 JJ=JJ-1                                                           DELB
C                                                                       DELB
C     INSERT ZEROS IN FIRST MU ROWS (NOT NECESSARY IN CASE MZ=0)        DELB
      IF(MZ)14,14,10                                                    DELB
   10 JJ=1                                                              DELB
      J=1+MZ                                                            DELB
      IC=1+MUD                                                          DELB
      DO 13 I=1,MU                                                      DELB
      DO 12 K=1,MC                                                      DELB
      A(JJ)=0.D0                                                        DELB
      IF(K-IC)11,11,12                                                  DELB
   11 A(JJ)=A(J)                                                        DELB
      J=J+1                                                             DELB
   12 JJ=JJ+1                                                           DELB
   13 IC=IC+1                                                           DELB
C                                                                       DELB
C     GENERATE TEST VALUE FOR SINGULARITY                               DELB
   14 TOL=EPS*PIV                                                       DELB
C                                                                       DELB
C                                                                       DELB
C     START DECOMPOSITION LOOP                                          DELB
      KST=1                                                             DELB
      IDST=MC                                                           DELB
      IC=MC-1                                                           DELB
      DO 26 K=1,M                                                       DELB
      IF(K-MR-1)16,16,15                                                DELB
   15 IDST=IDST-1                                                       DELB
   16 ID=IDST                                                           DELB
      ILR=K+MLD                                                         DELB
      IF(ILR-M)18,18,17                                                 DELB
   17 ILR=M                                                             DELB
   18 II=KST                                                            DELB
C                                                                       DELB
C     PIVOT SEARCH IN FIRST COLUMN (ROW INDEXES FROM I=K UP TO I=ILR)   DELB
      PIV=0.D0                                                          DELB
      DO 22 I=K,ILR                                                     DELB
      TB=DABS(A(II))                                                    DELB
      IF(TB-PIV)20,20,19                                                DELB
   19 PIV=TB                                                            DELB
      J=I                                                               DELB
```

```
      JJ=II                                                          DELB
   20 IF(I-MR)22,22,21                                               DELB
   21 ID=ID-1                                                        DELB
   22 II=II+ID                                                       DELB
C                                                                    DELB
C     TEST ON SINGULARITY                                            DELB
      IF(PIV)47,47,23                                                DELB
   23 IF(IER)26,24,26                                                DELB
   24 IF(PIV-TOL)25,25,26                                            DELB
   25 IER=K-1                                                        DELB
   26 PIV=1.DO/A(JJ)                                                 DELB
C                                                                    DELB
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R    DELB
      ID=J-K                                                         DELB
      DO 27 I=K,NM,M                                                 DELB
      II=I+ID                                                        DELB
      TB=PIV*R(II)                                                   DELB
      R(II)=R(I)                                                     DELB
   27 R(I)=TB                                                        DELB
C                                                                    DELB
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN COEFFICIENT MATRIX A DELB
      II=KST                                                         DELB
      J=JJ+IC                                                        DELB
      DO 28 I=JJ,J                                                   DELB
      TB=PIV*A(I)                                                    DELB
      A(I)=A(II)                                                     DELB
      A(II)=TB                                                       DELB
   28 II=II+1                                                        DELB
C                                                                    DELB
C     ELEMENT REDUCTION                                              DELB
      IF(K-ILR)29,34,34                                              DELB
   29 ID=KST                                                         DELB
      II=K+1                                                         DELB
      MU=KST+1                                                       DELB
      MZ=KST+IC                                                      DELB
      DO 33 I=II,ILR                                                 DELB
C                                                                    DELB
C     IN MATRIX A                                                    DELB
      ID=ID+MC                                                       DELB
      JJ=I-MR-1                                                      DELB
      IF(JJ)31,31,30                                                 DELB
   30 ID=ID-JJ                                                       DELB
   31 PIV=-A(ID)                                                     DELB
      J=ID+1                                                         DELB
      DO 32 JJ=MU,MZ                                                 DELB
      A(J-1)=A(J)+PIV*A(JJ)                                          DELB
   32 J=J+1                                                          DELB
      A(J-1)=0.DO                                                    DELB
C                                                                    DELB
C     IN MATRIX R                                                    DELB
      J=K                                                            DELB
      DO 33 JJ=I,NM,M                                                DELB
```

```
208          R(JJ)=R(JJ)+PIV*R(J)                                          DELB
209       33 J=J+M                                                         DELB
210       34 KST=KST+MC                                                    DELB
211          IF(ILR-MR)36,35,35                                            DELB
212       35 IC=IC-1                                                       DELB
213       36 ID=K-MR                                                       DELB
214          IF(ID)38,38,37                                                DELB
215       37 KST=KST-ID                                                    DELB
216       38 CONTINUE                                                      DELB
217    C     END OF DECOMPOSITION LOOP                                     DELB
218    C                                                                   DELB
219    C                                                                   DELB
220    C     BACK SUBSTITUTION                                             DELB
221          IF(MC-1)46,46,39                                              DELB
222       39 IC=2                                                          DELB
223          KST=MA+ML-MC+2                                                DELB
224          II=M                                                          DELB
225          DO 45 I=2,M                                                   DELB
226          KST=KST-MC                                                    DELB
227          II=II-1                                                       DELB
228          J=II-MR                                                       DELB
229          IF(J)41,41,40                                                 DELB
230       40 KST=KST+J                                                     DELB
231       41 DO 43 J=II,NM,M                                               DELB
232          TB=R(J)                                                       DELB
233          MZ=KST+IC-2                                                   DELB
234          ID=J                                                          DELB
235          DO 42 JJ=KST,MZ                                               DELB
236          ID=ID+1                                                       DELB
237       42 TB=TB-A(JJ)*R(ID)                                             DELB
238       43 R(J)=TB                                                       DELB
239          IF(IC-MC)44,45,45                                             DELB
240       44 IC=IC+1                                                       DELB
241       45 CONTINUE                                                      DELB
242       46 RETURN                                                        DELB
243    C                                                                   DELB
244    C                                                                   DELB
245    C     ERROR RETURN                                                  DELB
246       47 IER=-1                                                        DELB
247          RETURN                                                        DELB
248          END

@PRT,S INVERT.DGELS
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```
MORFITT*INVERT(1).DGELS
      1          SUBROUTINE DGELS(R,A,M,N,EPS,IER,AUX)                                DELS
      2    C                                                                          DELS
      3    C     ..........................................................................DELS
      4    C                                                                          DELS
      5    C        SUBROUTINE DGELS                                                  DELS
      6    C                                                                          DELS
      7    C        PURPOSE                                                           DELS
      8    C           TO SOLVE A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS WITH        DELS
      9    C           SYMMETRIC COEFFICIENT MATRIX UPPER TRIANGULAR PART OF WHICH    DELS
     10    C           IS ASSUMED TO BE STORED COLUMNWISE.                            DELS
     11    C                                                                          DELS
     12    C        USAGE                                                             DELS
     13    C           CALL DGELS(R,A,M,N,EPS,IER,AUX)                                DELS
     14    C                                                                          DELS
     15    C        DESCRIPTION OF PARAMETERS                                         DELS
     16    C           R       - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX       DELS
     17    C                     (DESTROYED). ON RETURN R CONTAINS THE SOLUTION OF    DELS
     18    C                     THE EQUATIONS.                                       DELS
     19    C           A       - UPPER TRIANGULAR PART OF THE SYMMETRIC DOUBLE        DELS
     20    C                     PRECISION M BY M COEFFICIENT MATRIX.  (DESTROYED)    DELS
     21    C           M       - THE NUMBER OF EQUATIONS IN THE SYSTEM.               DELS
     22    C           N       - THE NUMBER OF RIGHT HAND SIDE VECTORS.               DELS
     23    C           EPS     - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS     DELS
     24    C                     RELATIVE TOLERANCE FOR TEST ON LOSS OF               DELS
     25    C                     SIGNIFICANCE.                                        DELS
     26    C           IER     - RESULTING ERROR PARAMETER CODED AS FOLLOWS           DELS
     27    C                     IER=0  - NO ERROR,                                   DELS
     28    C                     IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR       DELS
     29    C                              PIVOT ELEMENT AT ANY ELIMINATION STEP       DELS
     30    C                              EQUAL TO 0,                                 DELS
     31    C                     IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-    DELS
     32    C                              CANCE INDICATED AT ELIMINATION STEP K+1,    DELS
     33    C                              WHERE PIVOT ELEMENT WAS LESS THAN OR        DELS
     34    C                              EQUAL TO THE INTERNAL TOLERANCE EPS TIMES   DELS
     35    C                              ABSOLUTELY GREATEST MAIN DIAGONAL           DELS
     36    C                              ELEMENT OF MATRIX A.                        DELS
     37    C           AUX     - DOUBLE PRECISION AUXILIARY STORAGE ARRAY             DELS
     38    C                     WITH DIMENSION M-1.                                  DELS
     39    C                                                                          DELS
     40    C        REMARKS                                                           DELS
     41    C           UPPER TRIANGULAR PART OF MATRIX A IS ASSUMED TO BE STORED      DELS
     42    C           COLUMNWISE IN M*(M+1)/2 SUCCESSIVE STORAGE LOCATIONS, RIGHT    DELS
     43    C           HAND SIDE MATRIX R COLUMNWISE IN N*M SUCCESSIVE STORAGE        DELS
     44    C           LOCATIONS. ON RETURN SOLUTION MATRIX R IS STORED COLUMNWISE     DELS
     45    C           TOO.                                                           DELS
     46    C           THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS    DELS
     47    C           GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS     DELS
     48    C           ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -       DELS
     49    C           INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL     DELS
     50    C           SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE    DELS
     51    C           INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS        DELS
```

```
C            GIVEN IN CASE M=1.                                              DELS
C            ERROR PARAMETER IER=-1 DOES NOT NECESSARILY MEAN THAT           DELS
C            MATRIX A IS SINGULAR, AS ONLY MAIN DIAGONAL ELEMENTS            DELS
C            ARE USED AS PIVOT ELEMENTS. POSSIBLY SUBROUTINE DGELG (WHICHDELS
C            WORKS WITH TOTAL PIVOTING) WOULD BE ABLE TO FIND A SOLUTION.DELS
C                                                                            DELS
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                      DELS
C            NONE                                                            DELS
C                                                                            DELS
C         METHOD                                                             DELS
C            SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH             DELS
C            PIVOTING IN MAIN DIAGONAL, IN ORDER TO PRESERVE                 DELS
C            SYMMETRY IN REMAINING COEFFICIENT MATRICES.                     DELS
C                                                                            DELS
C     ..........................................................................DELS
C                                                                            DELS
C                                                                            DELS
C                                                                            DELS
      DIMENSION A(1),R(1),AUX(1)                                             DELS
      DOUBLE PRECISION R,A,AUX,PIV,TB,TOL,PIVI                               DELS
      IF(M)24,24,1                                                           DELS
C                                                                            DELS
C     SEARCH FOR GREATEST MAIN DIAGONAL ELEMENT                              DELS
    1 IER=0                                                                  DELS
      PIV=0.D0                                                               DELS
      L=0                                                                    DELS
      DO 3 K=1,M                                                             DELS
      L=L+K                                                                  DELS
      TB=DABS(A(L))                                                          DELS
      IF(TB-PIV)3,3,2                                                        DELS
    2 PIV=TB                                                                 DELS
      I=L                                                                    DELS
      J=K                                                                    DELS
    3 CONTINUE                                                               DELS
      TOL=EPS*PIV                                                            DELS
C     MAIN DIAGONAL ELEMENT A(I)=A(J,J) IS FIRST PIVOT ELEMENT.              DELS
C     PIV CONTAINS THE ABSOLUTE VALUE OF A(I).                               DELS
C                                                                            DELS
C                                                                            DELS
C     START ELIMINATION LOOP                                                 DELS
      LST=0                                                                  DELS
      NM=N*M                                                                 DELS
      LEND=M-1                                                               DELS
      DO 18 K=1,M                                                            DELS
C                                                                            DELS
C     TEST ON USEFULNESS OF SYMMETRIC ALGORITHM                              DELS
      IF(PIV)24,24,4                                                         DELS
    4 IF(IER)7,5,7                                                           DELS
    5 IF(PIV-TOL)6,6,7                                                       DELS
    6 IER=K-1                                                                DELS
    7 LT=J-K                                                                 DELS
      LST=LST+K                                                              DELS
```

```
C                                                                       DELS
C       PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R     DELS
        PIVI=1.D0/A(I)                                                  DELS
        DO 8 L=K,NM,M                                                   DELS
        LL=L+LT                                                         DELS
        TB=PIVI*R(LL)                                                   DELS
        R(LL)=R(L)                                                      DELS
      8 R(L)=TB                                                         DELS
C                                                                       DELS
C       IS ELIMINATION TERMINATED                                       DELS
        IF(K-M)9,19,19                                                  DELS
C                                                                       DELS
C       ROW AND COLUMN INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A. DELS
C       ELEMENTS OF PIVOT COLUMN ARE SAVED IN AUXILIARY VECTOR AUX.     DELS
      9 LR=LST+(LT*(K+J-1))/2                                           DELS
        LL=LR                                                           DELS
        L=LST                                                           DELS
        DO 14 II=K,LEND                                                 DELS
        L=L+II                                                          DELS
        LL=LL+1                                                         DELS
        IF(L-LR)12,10,11                                                DELS
     10 A(LL)=A(LST)                                                    DELS
        TB=A                                                            DELS
        GO TO 13                                                        DELS
     11 LL=L+LT                                                         DELS
     12 TB=A(LL)                                                        DELS
        A(LL)=A(L)                                                      DELS
     13 AUX(II)=TB                                                      DELS
     14 A(L)=PIVI*TB                                                    DELS
C                                                                       DELS
C       SAVE COLUMN INTERCHANGE INFORMATION                             DELS
        A(LST)=LT                                                       DELS
C                                                                       DELS
C       ELEMENT REDUCTION AND SEARCH FOR NEXT PIVOT                     DELS
        PIV=0.D0                                                        DELS
        LLST=LST                                                        DELS
        LT=0                                                            DELS
        DO 18 II=K,LEND                                                 DELS
        PIVI=-AUX(II)                                                   DELS
        LL=LLST                                                         DELS
        LT=LT+1                                                         DELS
        DO 15 LLD=II,LEND                                               DELS
        LL=LL+LLD                                                       DELS
        L=LL+LT                                                         DELS
     15 A(L)=A(L)+PIVI*A(LL)                                            DELS
        LLST=LLST+II                                                    DELS
        LR=LLST+LT                                                      DELS
        TB=DABS(A(LR))                                                  DELS
        IF(TB-PIV)17,17,16                                              DELS
     16 PIV=TB                                                          DELS
        I=LR                                                            DELS
        J=II+1                                                          DELS
```

```
156       17 DO 1o LR=K,NM,M                                              DELS
157          LL=LR+LT                                                     DELS
158       18 R(LL)=R(LL)+PIVI*R(LR)                                       DELS
159    C     END OF ELIMINATION LOOP                                      DELS
160    C                                                                  DELS
161    C                                                                  DELS
162    C     BACK SUBSTITUTION AND BACK INTERCHANGE                       DELS
163       19 IF(LEND)24,23,2C                                             DELS
164       20 II=M                                                         DELS
165          DO 22 I=2,M                                                  DELS
166          LST=LST-II                                                   DELS
167          II=II-1                                                      DELS
168          L=A(LST)+.5D0                                                DELS
169          DO 22 J=II,NM,M                                              DELS
170          TB=R(J)                                                      DELS
171          LL=J                                                         DELS
172          K=LST                                                        DELS
173          DO 21 LT=II,LEND                                             DELS
174          LL=LL+1                                                      DELS
175          K=K+LT                                                       DELS
176       21 TB=TB-A(K)*R(LL)                                             DELS
177          K=J+L                                                        DELS
178          R(J)=R(K)                                                    DELS
179       22 R(K)=TB                                                      DELS
180       23 RETURN                                                       DELS
181    C                                                                  DELS
182    C                                                                  DELS
183    C     ERROR RETURN                                                 DELS
184       24 IER=-1                                                       DELS
185          RETURN                                                       DELS
186          END

&PRT,S INVERT.CLINEQ
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```
MORFITT*INVERT(1).CLINEQ
      1         SUBROUTINE CLIN EQ (A, B, X, N,                          00000
      2        $   N DIM, IFLAG, ERR)                                    00000
      3  C                                                               00000
      4  C    CLIN EQ USES L-U DECOMPOSITION TO                          00000
      5  C    FIND THE TRIANGULAR MATRICES L, U                          00000
      6  C    SUCH THAT  L * U = A.  L AND U ARE                         00000
      7  C    STORED IN A.  THIS FORM IS USED WITH                       00000
      8  C    BACK-SUBSTITUTION TO FIND THE SOLN                         00000
      9  C    X OF    A * X = L * U * X = B.                             00000
     10  C    N IS THE NUMBER OF EQUATIONS AND                           00001
     11  C    N DIM IS THE DIMENSION OF ALL ARRAYS                       00001
     12  C    IN THE PARAMETER LIST.                                     00001
     13  C                                                               00001
     14  C    IF IFLAG = 0, L, U, AND X ARE                              00001
     15  C    COMPUTED.                                                  00001
     16  C    IF IFLAG IS NON-ZERO, IT IS ASSUMED                        00001
     17  C    THAT L AND U HAVE BEEN COMPUTED IN                         00001
     18  C    A PREVIOUS CALL AND ARE STILL STORED                       00001
     19  C    IN A.  THUS ONLY X IS COMPUTED.                            00001
     20  C    ERR  IS THE ESTIMATED RELATIVE                             00002
     21  C    ERROR OF THE SOLUTION VECTOR.                              00002
     22  C                                                               00002
     23         COMPLEX*16  A, B, X, T                                   00002
     24  C      INTEGER*2  IROW                                          TJM02
     25         INTEGER*4  IROW                                          TJM02
     26         DIMENSION  A(N DIM, N DIM),                              00002
     27        $  B(N DIM), X(N DIM)                                     00002
     28         DIMENSION  IROW(50), Q(50)                               00002
     29         DATA  EPS /1.0E-15/                                      00002
     30  C                                                               00002
     31  C                                                               00003
     32         IF (N.GT.50)  GO TO 900                                  00003
     33         IF (IFLAG.NE.0)  GO TO 600                               00003
     34         DO 050  I = 1,N                                          00003
     35         Q(I) = 0.0                                               00003
     36         DO 040  J = 1,N                                          00003
     37         QQ = CDABS (A(I,J))                                      00003
     38     040 IF (Q(I).LT.QQ)  Q(I) = QQ                               00003
     39         IF (Q(I).EQ.0.0)  GO TO 901                              00003
     40     050 CONTINUE                                                 00003
     41         ERR = EPS                                                00004
     42         PPIV = 0.0                                               00004
     43         DO 100  I = 1,N                                          00004
     44     100 IROW(I) = I                                              00004
     45  C                                                               00004
     46         DO 500  L = 1,N                                          00004
     47         PIVOT = 0.0                                              00004
     48         K = L - 1                                                00004
     49         DO 240  I = L,N                                          00004
     50         IF (K.LT.1)  GO TO 230                                   00004
     51         DO 220  J = 1,K                                          00005
```

```
    220 A(I,L) = A(I,L) - A(J,L) * A(I,J)                                 0000
    230 F = CDABS (A(I,L)) / Q(I)                                         0000
        IF (PIVOT.GT.F)  GO TO 240                                        0000
        PIVOT = F                                                         0000
        NPIVOT = I                                                        0000
    240 CONTINUE                                                          0000
        IF (PIVOT.EQ.0.0)  GO TO 901                                      0000
        IF (PPIV.LE.PIVOT)  GO TO 250                                     0000
        ERR = ERR * PPIV / PIVOT                                          0000
        IF (ERR.GE.1.0)  GO TO 901                                        0000
    250 PPIV = PIVOT                                                      0000
        IF (NPIVOT.EQ.L)  GO TO 280                                       0000
        Q(NPIVOT) = Q(L)                                                  0000
        J = IROW(L)                                                       0000
        IROW(L) = IROW(NPIVOT)                                            0000
        IROW(NPIVOT) = J                                                  0000
        DO 260  I = 1,N                                                   0000
        T = A(L,I)                                                        0000
        A(L,I) = A(NPIVOT,I)                                              0000
        A(NPIVOT,I) = T                                                   0000
    260 CONTINUE                                                          0000
    280 IF (L.EQ.N)  GO TO 500                                            0000
        T = (1.0D0,0.0D0) / A(L,L)                                        0000
        K = L + 1                                                         0000
        M = L - 1                                                         0000
        DO 450  I = K,N                                                   0000
        IF (M.LT.1)  GO TO 400                                            0000
        DO 350  J = 1,M                                                   0000
    350 A(L,I) = A(L,I) - A(L,J) * A(J,I)                                 0000
    400 A(L,I) = T * A(L,I)                                               0000
    450 CONTINUE                                                          0000
    500 CONTINUE                                                          0000
        IF (ERR.GT.1.0E-5)    PRINT 998, ERR                              0000
C                                                                         0000
C                                                                         0000
    600 DO 620  I = 2,N                                                   0000
    620 X(I) = (0.0D0,0.0D0)                                              0000
        J = IROW(1)                                                       0000
        X(1) = B(J) / A(1,1)                                              0000
        DO 700  I = 2,N                                                   0000
        J = IROW(I)                                                       0000
        K = I - 1                                                         0000
        DO 650  L = 1,K                                                   0000
    650 X(I) = X(I) + A(I,L) * X(L)                                       0000
        X(I) = (B(J) - X(I)) / A(I,I)                                     0000
    700 CONTINUE                                                          0000
        K = N - 1                                                         0000
        DO 800  I = 1,K                                                   0000
        J = N - I                                                         0001
        M = J + 1                                                         0001
        DO 800  L = M,N                                                   0001
        X(J) = X(J) - X(L) * A(J,L)                                       0001
```

```
      800 CONTINUE                                                      0001
          RETURN                                                        0001
      C                                                                 0001
      900 PRINT 999                                                     0001
          ERR = 1.0                                                     0001
          RETURN                                                        0001
      901 PRINT 997                                                     0001
          ERR = 1.0                                                     0001
          RETURN                                                        0001
      997 FORMAT ('1ERROR IN CLIN EQ, MATRIX IS SINGULAR')              0001
      998 FORMAT (' CAUTION-',                                          0001
     $ ' CLIN EQ HAS DECOMPOSED AN ILL-CONDITIONED MATRIX.',/,          0001
     $ ' RESULTS WILL HAVE RELATIVE ERROR =',E11.2)                     0001
      999 FORMAT ('1ERROR IN CLIN EQ, MATRIX SIZE GREATER THAN 50')     0001
          END                                                           0001

@PRT,S INVERT.OVCHK
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```
MORFITT*INVERT(1).OVCHK
      1         SUBROUTINE OVCHK(CALLER,STMT)                                  DMP
      2   C ***** THIS SUBROUTINE WILL CALL THE ROUTINE OVUNFL WHICH           DMP
      3   C ***** CHECKS FOR OVERFLOW AND UNDERFLOW.  THE ROUTINE RETURNS AN   DMP
      4   C ***** INTEGER VALUE WHICH TELLS WHETHER OVERFLOW AND/OR UNDERFLOW  DMP
      5   C ***** HAVE OCCURRED SINCE THE PROGRAM STARTED OR SINCE THE LAST TIME DMP
      6   C ***** ONE OF THE OVERFLOW/UNDERFLOW CHECKING ROUTINES WAS CALLED,  DMP
      7   C ***** WHICHEVER HAS BEEN MORE RECENT. THE INTEGER VALUES RETURNED  DMP
      8   C ***** AND THEIR MEANINGS ARE AS FOLLOWS:                          DMP
      9   C *****       1= OVERFLOW ONLY                                      DMP
     10   C *****       2= NO OVERFLOW OR UNDERFLOW                           DMP
     11   C *****       3= UNDERFLOW ONLY                                     DMP
     12   C *****       4= OVERFLOW AND UNDERFLOW                             DMP
     13   C ***** THIS SUBROUTINE (OVCHK) WILL PRINT OUT A MESSAGE STATING    DMP
     14   C ***** WHETHER OVERFLOW HAS OR HAS NOT OCCURRED. FOR PURPOSES      DMP
     15   C ***** OF THIS CONVERSION, UNDERFLOW OCCURRENCES HAVE BEEN IGNORED  DMP
     16   C ***** BECAUSE THE 1110 TAKES THE SAME ACTION (I.E. SETS THE RESULT DMP
     17   C ***** TO ZERO) IN THE CASE OF UNDERFLOW AS DOES THE 360.          DMP
     18   C ***** THE SUBROUTINE ALSO PRINTS OUT THE NAME OF THE CALLING      DMP
     19   C ***** ROUTINE (VARIABLE 'CALLER') AND THE STATEMENT NUMBER FROM    DMP
     20   C ***** WHICH IT WAS CALLED (VARIABLE 'STMT')                       DMP
     21         CHARACTER*6 CALLER                                             DMP
     22         INTEGER STMT                                                   DMP
     23         CALL OVUNFL(IOVFL)                                             DMP
     24         IF (IOVFL .EQ. 1 .OR. IOVFL .EQ. 4) GO TO 150                  DMP
     25         WRITE (6,100)                                                  DMP
     26     100 FORMAT ('1OVERFLOW HAS NOT OCCURRED')                          DMP
     27         GO TO 250                                                      DMP
     28     150 WRITE (6,200)                                                  DMP
     29     200 FORMAT ('1OVERFLOW HAS OCCURRED')                              DMP
     30     250 CONTINUE                                                       DMP
     31         WRITE (6,300) STMT,CALLER                                      DMP
     32     300 FORMAT (' OVCHK WAS CALLED FROM STATEMENT ',I5,' IN SUBROUTINE DMP
     33        1   ,A6)                                                       DMP
     34         RETURN                                                         DMP
     35         END                                                            DMP


$FIN
```